Volume of Rectangular Solids and Cylinders

In the last lesson, we learned that the area of a rectangle is the number of unit squares inside the rectangle.

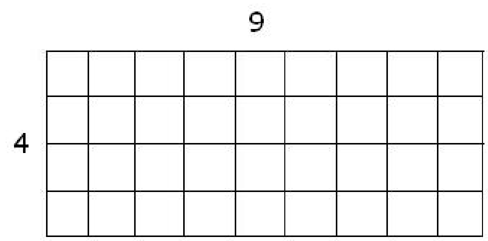

Figure 1: A 4-by-9 rectangle with 36 unit squares inside

Similarly, the volume of a solid is the number of unit cubes inside the solid.

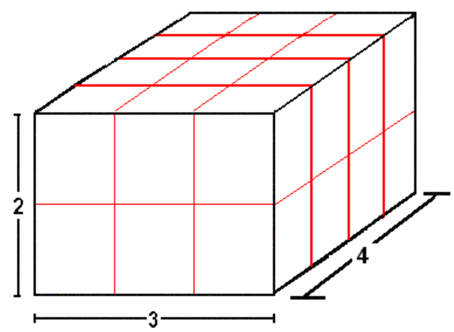

Figure 2: A 3-by-4-by-2 rectangular solid

Figure 2 shows a straight rectangular solid (also called "prism"). Notice that the solid is made up of 1x1x1 unit cubes. The bottom layer has $3 \cdot 4 = 12$ such unit cubes. With 2 layers, the solid has a total of $3 \cdot 4 \cdot 2 = 24$ unit cubes, implying the solid's volume is 24 cubic units.

In summary, to find this prism's volume, we first found the area of the base by $3 \cdot 4 = 12$. Next, we multiplied the base area by the prism's height, $3 \cdot 4 \cdot 2 = 24$, which gives us the prism's volume.

We can use the same method to find the volume of any prism: $\text{volume} = (\text{base area}) \cdot (\text{height})$

[**Example 1**] Find the volume of a cube, whose side length is 5 cm.

[**Solution**] A cube is straight, so we can use the formula: $\text{volume} = (\text{base area}) \cdot (\text{height})$

The base of a cube is a square, so it's area is $5 \cdot 5 = 25$ cm^2.

Next, the cube's volume is $(\text{base area}) \cdot (\text{height})$: $25 \cdot 5 = 125$ cm^3.

Solution: The cube's volume is 125 cm^3. Note that "cm^3" means cubic centimeters.

A cylinder is straight, so we can still use the formula $\text{volume} = (\text{base area}) \cdot (\text{height})$.

[**Example 2**] A cylinder's height is 10 inches. Its base's radius is 3 inches. Find the cylinder's volume in terms of π.

[**Solution**] Since a cylinder is straight, we can use the formula $\text{volume} = (\text{base area}) \cdot (\text{height})$.

A cylinder's base is a circle. We calculate its area by a circle's area formula:

$$\text{base area} = \pi \cdot r^2 = \pi \cdot 3^2 = 9\pi \ \text{in}^2$$

Note that we didn't change the result to decimal because the problem asks for an exact value in terms of π.

Next, we can find the cylinder's volume:

$$\text{volume} = (\text{base area}) \cdot (\text{height}) = 9\pi \cdot 10 = 90\pi \ \text{in}^3$$

Solution: The cylinder's volume is $90\pi \ \text{in}^3$. Notice the differences in units:

- in represents "inch" in length;
- in^2 represents "square inch" in area;
- in^3 represents "cubic inch" in volume.

Since we have learned how to solve linear equations, we can use formulas "backward".

[**Example 3**] A rectangular prism's volume is 128 ft^3. Its base is a 5-ft by 4-ft rectangle. Find this prism's height.

[**Solution**] Let the prism's height be h ft long.

Substitute given numbers into the formula $\text{volume} = (\text{base area}) \cdot (\text{height})$, we have:

$$\begin{aligned}128 &= 5 \cdot 4 \cdot h \\ 128 &= 20h \\ \frac{128}{20} &= \frac{20h}{20} \\ 6.4 &= h\end{aligned}$$

Solution: The prism's height is 6.4 ft.

We will finish this lesson with a slightly challenging example.

[**Example 4**] A cylinder's volume is 900 mm^3. Its height is 10 mm. Find the radius of its base. Round your answers to two decimal places.

[**Solution**] Let the base's radius be r mm.

By the formula $\text{volume} = (\text{base area}) \cdot (\text{height})$, we have:

$$volume = \pi \cdot r^2 \cdot h$$
$$900 = \pi \cdot r^2 \cdot 10$$
$$\frac{900}{10\pi} = \frac{\pi \cdot r^2 \cdot 10}{10\pi}$$
$$\frac{900}{10\pi} = r^2$$
$$\frac{900}{10 \cdot 3.14159} \approx r^2$$
$$28.64789 \approx r^2$$
$$\sqrt{28.64789} \approx r$$
$$5.35 \approx r$$

Solution: The cylinder's base's radius is approximately 5.35 mm.