Subsets of Real Numbers

There are a few categories of real numbers. You need to know a certain number belongs to which category.

Natural Numbers

Natural numbers are also called counting numbers. At the very beginning of civilization, we only needed to count 1, 2, 3, ...

Whole Numbers

Later, people needed a number to represent nothing, so 0 was developed. We call the set {0, 1, 2, 3, ...} whole numbers. Note that whole numbers include all natural numbers.

Integers

Then, people started to owe others money. To represent the concept of "owing," negative numbers were developed. We call the set {..., -3, -2, -1, 0, 1, 2, 3, ...} integers.

Rational Numbers

What if someone needs one and a half of something? People need decimals and fractions. If a number can be represented by $\frac{a}{b}$, where a and b are integers, this number belongs to the rational numbers category. Rational numbers include all previous categories. Here are some examples:

- The natural number 4 is also a rational number because $4 = \frac{4}{1}$.
- The whole number 0 is also a rational number because $0 = \frac{0}{1}$.
- The integer -3 is also a rational number because $-3 = \frac{-3}{1}$.
- The decimal 0.123 is a rational number because $0.123 = \frac{123}{1000}$.
- The repeating decimal $0.\overline{12}$ (or $0.12121212\ldots$) is a rational number because $0.\overline{12} = \frac{12}{99}$ (verify this with a calculator).
- The repeating decimal $0.45\overline{12}$ (or $0.4512121212\ldots$) is a rational number because $0.45\overline{12} = \frac{1489}{3300}$ (again, verify this with a calculator).

Irrational Numbers

If a number cannot be represented by $\frac{a}{b}$, where a and b are integers, this number is an irrational number. The constant π is such a number, as it never ends, and it doesn't have a repeating pattern.

Note that the decimal $0.\overline{12}$ and $0.45\overline{12}$ also never ends, but they have a repeating pattern. A decimal with a repeating pattern can be written as a fraction, as shown above.

Number like $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$ are irrational numbers. Notice that $\sqrt{4}$ is not in this list, because 4 is a square number, and $\sqrt{4}=2$. Be very careful here: $\sqrt{0}$, $\sqrt{1}$, $\sqrt{4}$, $\sqrt{9}$ and the square root of all square numbers are rational number (actually whole numbers).

Real Numbers

Finally, all categories above, from natural numbers to irrational numbers, belong to the real numbers category. In this course, we only deal with real numbers. In later courses, we will learn non-real numbers.

Number Line Revisited

Now we need to locate not only integers, but also other types of numbers on the number line. Here are a few examples:

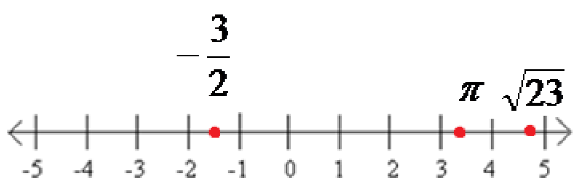

Figure 1: -3/2, PI and sqrt(23) on the number line

To locate $-\frac{3}{2}$, we change it to decimal: $-\frac{3}{2}=-3\div 2=-1.5$.

You must memorize that $\pi = 3.1415926...$

Because $\sqrt{25}=5$, we know $\sqrt{23}$ must be very close to 5. The calculator tells us $\sqrt{23}\approx 4.80$