Three Exponent Rules

Earlier we learned the definition of exponent: $2^3 = 2 \cdot 2 \cdot 2$

Today we will learn 3 exponent rules.

Product Rule: $x^a \cdot x^b = x^{a+b}$

The way to learn rules is not to memorize formulas. Otherwise, tomorrow you will forget. The key is understanding. If you understand why a rule works that way, you can re-produce the rule on scratch paper any time you want.

Let's look at an example:

$$x^2 \cdot x^3 = x^?$$

The key to do exponent problems boils down to one word: expand.

We expand x^2 and x^3, and we have:

$$x^2 \cdot x^3 = (x \cdot x) \cdot (x \cdot x \cdot x)$$

Now it's clear the answer is:

$$x^2 \cdot x^3 = (x \cdot x) \cdot (x \cdot x \cdot x) = x^5$$

This is why we add the exponent in the rule $x^a \cdot x^b = x^{a+b}$.

[**Example 1**] $2^{10} \cdot 2^{20} = 2^{30}$

Note that $2^2 \cdot 3^3$ cannot be combined into one exponent, because the bases are different.

[**Example 2**] $2^{10} \cdot 2 = 2^{11}$

Remember that if a number doesn't have any exponent, it implies "to the first power". In other words, $2 = 2^1$

Scroll down for more notes.

Product to a Power Rule: $(xy)^a = x^a y^a$

Again, let's try to under this rule by "expanding." Here is an example:

$$(xy)^3 = ?$$

After we expand the exponent expression, we have:

$$(xy)^3 = (xy)(xy)(xy)$$

When numbers multiply each other, the order doesn't matter. For example:

$$2 \cdot 3 \cdot 4 = 24\ ,$$

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We reorganize the order in $(xy)(xy)(xy)$, and we have:

$$(xy)^3 = (xy)(xy)(xy) = xxxyyy = x^3 y^3$$

This is why the rule works like $(xy)^a = x^a y^a$.

[**Example 3**] $(2x)^4 = 2^4 x^4 = 16x^4$

For values smaller than 100, like 3^2, 2^3, 2^5, 4^3, you are expected to change them to numbers, like 2^4=16 in Example 2. If the number is too big, like 5^7, you can leave it as 5^7. Different instructors have different expectations on when you should change an exponent expression to its value.

Scroll down for more notes.

Power to a Power Rule: $(x^a)^b = x^{ab}$

Let's look at an example by the technique "expanding":

$$(x^2)^3 = (x^2)(x^2)(x^2) = (xx)(xx)(xx) = x^6$$

Note the difference between $(x^a)^b = x^{ab}$ and the first rule we learned: $x^a \cdot x^b = x^{a+b}$. As long as you understand how to expand an exponent expression, you will understand why these two rules are different, and there is no need to memorize these rules.

[**Example 4**] $(5^4)^{10} = 5^{40}$

Now, let's look at a few problems involving more than one of those 3 rules we just learned.

[**Example 5**] $(2^4 x)^{10} = (2^4)^{10} x^{10} = 2^{40} x^{10}$

[**Example 6**]

$$\begin{aligned} &(x^3 y^2)^5 y^{10} \\ &= (x^3)^5 (y^2)^5 y^{10} \\ &= x^{15} y^{10} y^{10} \\ &= x^{15} y^{20} \end{aligned}$$

[**Example 7**]

$$\begin{aligned} &(x^3 y^2 x)^5 \\ &= (x^4 y^2)^5 \\ &= (x^4)^5 (y^2)^5 \\ &= x^{20} y^{10} \end{aligned}$$

It's easier to combine x^3 and x into x^4 inside the parentheses first.