Standard Form Equation $Ax + By = C$

Let's introduce standard form equation in a scenario.

You have a budget of $24 to purchase some markers and some staplers. Each marker costs $2, and each stapler costs $3. Let *x* represent the number of markers you will purchase, and *y* represent the number of staplers you will purchase.

Since each marker costs $2, *x* markers will cost 2x dollars.

Since each stapler costs $3, *y* stapler will cost 3x dollars.

The total cost is 2*x*+3*y* dollars.

If all $24 in the budget is spent, we have the equation:

$$2x + 3y = 24$$

This equation looks different from the slope-intercept form $y = Mx + B$. We call equations in the form $2x + 3y = 24$ the **standard form** of a linear equation.

Note that the slope of $2x + 3y = 24$ is not 2. If a linear equation is not in slope-intercept form, the number in front of *x* is not the line's slope. How do we find the slope of $2x + 3y = 24$?

[**Example 1**] Find the slope and *y*-intercept of $2x + 3y = 24$.

[**Solution**] We need to change this line's equation from standard form to slope-intercept form, where the variable *y* is by itself on one side of the equal sign:

$$\begin{aligned} 2x + 3y &= 24 \\ 2x + 3y - 2x &= 24 - 2x \\ 3y &= -2x + 24 \\ \frac{3y}{3} &= \frac{-2x}{3} + \frac{24}{3} \\ y &= -\frac{2}{3}x + 8 \end{aligned}$$

Note that in Step 3, we switch $24 - 2x$ to $-2x + 24$, because it's a math convention to write terms with variables first.

Solution: Now we can tell the slope of $2x + 3y = 24$ is $-\frac{2}{3}$, and the *y*-intercept is (0, 8).

What does the slope and *y*-intercept mean in this scenario?

The y-intercept is easier to understand. Recall that *x* represents the number of markers, and *y* represents the number of staplers. The *y*-intercept (0, 8) means: We can spend \$24 to purchase 0 marker and 8 staplers. This makes sense since each stapler costs \$3.

The slope shows the relationship between *x* and *y*, or the number of markers and the number of staplers. The slope of $-\frac{2}{3}$ in this scenario means: For each 3 more markers we purchase, we can purchase 2 fewer staplers. This makes sense since each marker costs \$2, and each stapler costs \$3.

Understanding the slope in this scenario might be challenging. Graphing this line should help.

[**Example 2**] Graph $2x+3y=24$.

[**Solution**] In Example 1, we have changed this line's equation from standard form to slope-intercept form:

$$y=-\frac{2}{3}x+8$$

Now we can graph this line by its slope and *y*-intercept.

First, we plot the y-intercept (0, 8).

Next, since the slope is $-\frac{2}{3}$, we start from (0, 8), rise by -2 units (go down by 2 units) and then run by 3 units (go right by 3 units). We will reach the point (3, 6).

We could do this a few more times to find more points on the line.

Finally, connect all points and then extend both ways.

See the graph on the next page.

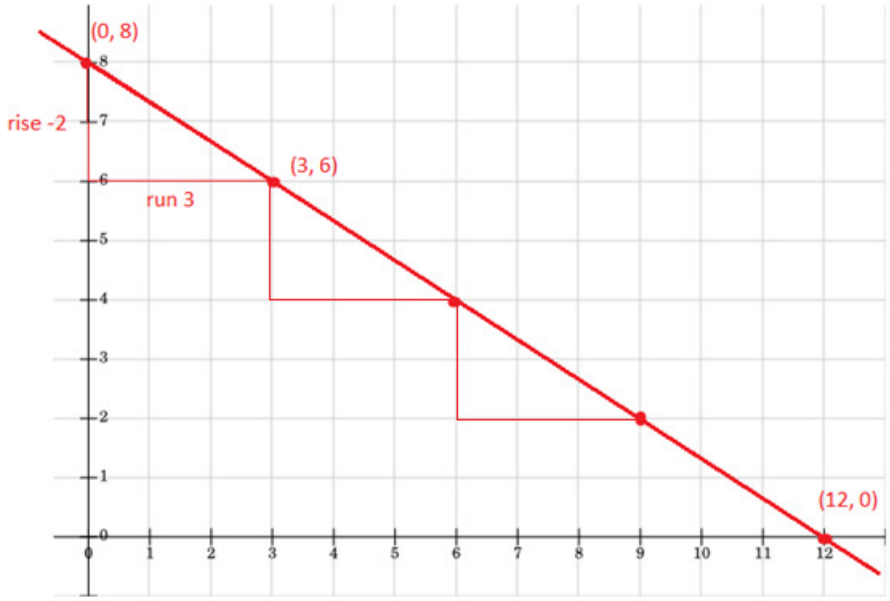

Figure 1: Graph of 2x+3y=24

Now let's try again to understand the slope, $-\frac{2}{3}$, in this scenario. Again, *x* represents the number of markers, and *y* represents the number of staplers.

The *y*-intercept, (0, 8), means we can use $24 to purchase 0 marker and 8 staplers.

The slope, $-\frac{2}{3}$, means the rate of change. In this scenario, it means for each 3 more markers we purchase, we can purchase 2 fewer staplers.

In the graph, if we start from (0, 8), rise -2 and then run 3, we would reach (3, 6). This point means we can purchase 3 markers and 6 staplers. Compared to (0, 8), we indeed purchased 3 more markers and 2 fewer staplers.

In this section, we learned one way to graph an equation given in standard form $Ax + By = C$:

We first change the equation from standard form to slope-intercept form, and then graph the line by its *y*-intercept and slope triangles.

Let's practice this skill. Later, we will learn how to graph $Ax + By = C$ by intercepts.