Equations with Special Solution Sets

There are two types of equations with special solution sets.

Equations with No Solution

[**Example 1**] Solve $x = x + 1$ for x.

[**Solution**]

$$\begin{aligned} x &= x + 1 \\ x - x &= x + 1 - x \\ 0 &= 1 \end{aligned}$$

Notice that both x terms disappeared at the same time! We've never seen this before. We cannot carry out Step 5—get rid of the number in front of the variable.

Let's think about the original equation, $x = x + 1$. Which number is equal to 1 plus the number itself? Nothing! Any number increased by 1 will become a different number. We say the equation $x = x + 1$ has no solution. Some textbooks say this equation has an empty solution set, denoted ϕ.

The result, 0=1, is false, so the equation $x = x + 1$ has no solutions. This means, no matter what number you plug into *x*, the equation $x = x + 1$ will never be true.

Compare Example 1 with $2x = x + 1$ and note the difference:

$$\begin{aligned} 2x &= x + 1 \\ 2x - x &= x + 1 - x \\ x &= 1 \end{aligned}$$

Here, x terms didn't disappear as in Example 1.

$2x = x + 1$ does have a solution: $x = 1$. Compare this with $x = x + 1$, which has no solution.

Equations with Infinitely Many Solutions (All Real Numbers)

[**Example 2**] Solve $x + 1 = x + 1$ for x.

[**Solution**]

$$\begin{aligned} x + 1 &= x + 1 \\ x + 1 - x &= x + 1 - x \\ 1 &= 1 \end{aligned}$$

Since 1=1 is true, the equation $x+1=x+1$ has infinitely many solutions, or its solution set is "all real numbers." This means, no matter what number you plug into *x*, the equation $x+1=x+1$ will always be true.

Summary

In summary, when we solve a linear equation, and if all variable terms disappeared, the equation either has "no solution" or "infinitely many solutions".

- If the result is false, like 0=1, the original equation has "no solution", or the solution set is empty.
- If the result is true, like 1=1, the original equation has infinitely many solutions, or the solution set is "all real numbers".