Special Case Inequalities

Recall that we learned how to solve special case equations. There are two types.

[**Example 1**] Solve $x = x + 1$ for x.

[**Solution**]

$$x = x + 1$$
$$x - x = x + 1 - x$$
$$0 = 1$$

Since 0=1 is false, the equation $x = x + 1$ has no solutions. This means, no matter what number you plug into *x*, the equation $x = x + 1$ will never be true.

[**Example 2**] Solve $x + 1 = x + 1$ for x.

[**Solution**]

$$x + 1 = x + 1$$
$$x + 1 - x = x + 1 - x$$
$$1 = 1$$

Since 1=1 is true, the equation $x + 1 = x + 1$ has infinitely many solutions, or "all real numbers." This means, no matter what number you plug into *x*, the equation $x + 1 = x + 1$ will always be true.

For inequalities, there are similar special cases. Example 3 has **no solution**.

[**Example 3**] Solve $x > x + 1$ for x.

[**Solution**]

$$x > x + 1$$
$$x - x > x + 1 - x$$
$$0 > 1$$

Since 0>1 is false, the inequality $x > x + 1$ has no solution. This means, no matter what number you plug into *x*, the inequality $x > x + 1$ will never be true.

Example 4 and Example 5 has **infinitely many solutions**, or **all real numbers**.

[**Example 4**] Solve $x \le x+1$ for x.

[**Solution**]

$$x \le x+1$$
$$x - x \le x+1-x$$
$$0 \le 1$$

Since $0 \le 1$ is true, the inequality $x \le x+1$ has infinitely many solutions, or "all real numbers." This means, no matter what number you plug into *x*, the inequality $x \le x+1$ will always be true.

[**Example 5**] Solve $\frac{x}{4} - 1 \le \frac{x+8}{4}$ for x.

[**Solution**]

$$\frac{x}{4} - 1 \le \frac{x+8}{4}$$
$$4 \cdot \frac{x}{4} - 4 \cdot 1 \le 4 \cdot \frac{x+8}{4}$$
$$x - 4 \le x + 8$$
$$x - 4 - x \le x + 8 - x$$
$$-4 \le 8$$

Since $-4 \le 8$ is true, the inequality $\frac{x}{4} - 1 \le \frac{x+8}{4}$ has infinitely many solutions, or "all real numbers."

This means, no matter what number you plug into *x*, the inequality $\frac{x}{4} - 1 \le \frac{x+8}{4}$ will always be true.