Solve Two-Step Linear Equations

Let's review those 5 steps to solve linear equations:

1. Get rid of parentheses by distributive property.
2. Combine like terms.
3. Move variable terms to one side of the equal sign.
4. Move number terms to the other side of the equal sign.
5. Get rid of the number in front of the variable.

Today, we learn how to solve equations with Step 3, 4, and 5.

[**Example 1**] Solve $2y+9=1$ for y.

[**Solution**] Notice that the only term with the variable *y* is on the left side of the equal sign, so Step 3 is not needed.

$2y+9=1$ Next step: Move number terms to the other side of equal sign.

$2y+9-9=1-9$

$2y=-8$ Next step: Get rid of the number in front of the variable.

$\frac{2y}{2}=\frac{-8}{2}$

$y=-4$

It's good practice to check the solution by plugging $y=-4$ into $2y+9=1$:

$$2y+9=1$$
$$2(-4)+9=1$$
$$-8+9=1$$
$$1=1$$

The solution checks. $y=-4$ is the solution of $2y+9=1$.

In Example 2, we use Step 3, 4 and 5.

[**Example 2**] Solve $2z-10=5z+14$ for z.

[**Solution**]

$2z-10=5z+14$ Next step: Move variable terms to one side of equal sign.

$2z-10-5z=5z+14-5z$

$-3z-10=14$ Next step: Move number terms to the other side of equal sign.

$-3z-10+10=14+10$

$-3z=24$ Next step: Get rid of number in front of the variable.

$\frac{-3z}{-3}=\frac{24}{-3}$

$z=-8$

Check the solution:

$$2z-10=5z+14$$
$$2(-8)-10=5(-8)+14$$
$$-16-10=-40+14$$
$$-26=-26$$

The solution checks! $z=-8$ is the solution of $2z-10=5z+14$.

Note that we could have moved variables to the right side of equal sign, and move number terms to the left side:

$$2z-10=5z+14$$
$$2z-10-2z=5z+14-2z$$
$$-10=3z+14$$
$$-10-14=3z+14-14$$
$$-24=3z$$
$$\frac{-24}{3}=\frac{3z}{3}$$
$$-8=z$$

This way, we avoided dealing with a negative number in front of z. Many students like the second method.

Example 3 shows a very common mistake.

[**Example 3**] Solve $4-m=10$ for m.

[**Solution**] First, let me show the mistake:

$$\begin{aligned} 4-m&=10\\ 4-m-4&=10-4\\ m&=6 \end{aligned}$$

Let's plug $m=6$ into $4-m=10$ to check:

$$\begin{aligned} 4-m&=10\\ 4-6&=10\\ -2&=10 \end{aligned}$$

Oops! What went wrong?

Note that the negative sign in front of m disappeared for no reason. That's the mistake!

Here is the correct solution:

$$\begin{aligned} 4-m&=10\\ 4-m-4&=10-4\\ -m&=6\\ (-1)\cdot(-m)&=(-1)\cdot 6\\ m&=-6 \end{aligned}$$

Let's check again by plugging in $m=-6$:

$$\begin{aligned} 4-m&=10\\ 4-(-6)&=10\\ 4+6&=10\\ 10&=10 \end{aligned}$$

$m=-6$ is the solution of $4-m=10$.

The lesson is: Don't throw away negative signs for no reason.

Example 4 shows another common mistake.

[**Example 4**] Solve $5-2n=11$ for n.

[**Solution**] Here is the wrong way to solve this equation:

$$\begin{aligned} 5-2n&=11\\ 5-2n+5&=11+5\\ -2n&=16\\ n&=-8 \end{aligned}$$

What went wrong?

To get rid of the number 5 on the left side, we need to subtract 5 on both sides, not adding 5. Otherwise, since 5+5=10, the number term won't disappear from the left side.

Here is the right way to solve Example 4:

$$\begin{aligned} 5-2n &= 11 \\ 5-2n-5 &= 11-5 \\ -2n &= 6 \\ n &= -3 \end{aligned}$$

If the equation were $-5-2n=11$, then we would add 5 on both sides.