Solve Proportions

In this lesson, we will learn how to solve proportions, like this equation: $\frac{3}{x}=\frac{4}{7}$.

Determine whether a proportion is true or false

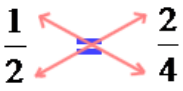
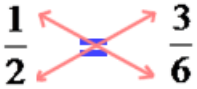
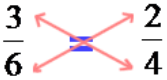
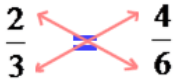
A proportion is a statement that two ratios or rates are equal. For instance, when we write:

$$\frac{3}{6}=\frac{1}{2}$$

We are saying that the ratio 3/6 is equal to the ratio 1/2. You know from your work with simplifying fractions that this is a true statement. However, when numbers are complicated, it's hard to tell, like this one:

$$\frac{4}{1.5}=\frac{8}{3}$$

We need to learn a skill called "**cross multiplication**." We start by observing a pattern:

$\frac{1}{2}=\frac{2}{4}$	$\frac{1}{2}=\frac{3}{6}$	$\frac{3}{6}=\frac{2}{4}$	$\frac{2}{3}=\frac{4}{6}$
			
$1\cdot 4=2\cdot 2$	$1\cdot 6=2\cdot 3$	$3\cdot 4=6\cdot 2$	$2\cdot 6=3\cdot 4$

Notice that if a proportion is true, the "cross product" is also true.

To judge whether $\frac{4}{1.5}=\frac{8}{3}$ is true, we will look at the cross products:

$$3\cdot 4=12,\ \text{ and }\ 1.5\cdot 8=12$$

This is why $\frac{4}{1.5}=\frac{8}{3}$ is true.

[**Example 1**] Is $\frac{5}{12}=\frac{4}{9}$ true?

[**Solution**] We check the cross product:

$$5\cdot 9=45,\ \text{ and }\ 12\cdot 4=48$$

Conclusion: $\frac{5}{12}=\frac{4}{9}$ is not true.

Demystify Cross Multiplication

We have learned enough knowledge not to memorize "cross multiplication" as a rule. We can understand why it works. Let's review two properties.

Property 1:

If we have an equation, say $1+2=3$, we can multiply each term in the equation by the same number, and the equation would still be true. For example, if we do $2\cdot 1+2\cdot 2=2\cdot 3$, we have $2+4=6$.

Property 2:

Review this shortcut method to multiply an integer with a fraction:

$$3\cdot\frac{2}{3}=3\div 3\cdot 2=2 \qquad 6\cdot\frac{2}{3}=6\div 3\cdot 2=4 \qquad 9\cdot\frac{2}{3}=9\div 3\cdot 2=6$$

When an integer is multiplied by a fraction, if the denominator goes into the integer, the product is always an integer.

Now we are ready to explain why cross multiplication works. Say we have:

$$\frac{4.5}{6}=\frac{4}{5}$$

To get rid of both fractions in this equation, we multiply both sides by 30, since both 6 and 5 go into 30. We have:

$$30 \cdot \frac{4.5}{6} = 30 \cdot \frac{4}{5}$$
$$30 \div 6 \cdot 4.5 = 30 \div 5 \cdot 4$$
$$5 \cdot 4.5 = 6 \cdot 4$$

This result is the same as if we did cross multiplication, and this is why cross multiplication works.

[**Example 2**] Solve $\frac{x}{3} = \frac{7}{15}$ for *x*.

[**Solution**] Using cross multiplication, we have:

$$\frac{x}{3} = \frac{7}{15}$$
$$15x = 3 \cdot 7$$
$$15x = 21$$
$$\frac{15x}{15} = \frac{21}{15}$$
$$x = \frac{7}{5}$$

In some context, you might change $\frac{7}{5}$ to 1.4

[**Example 3**] Solve $\frac{3}{x} = \frac{15}{7}$ for *x*.

[**Solution**] Using cross multiplication, we have:

$$\frac{3}{x} = \frac{15}{7}$$
$$15x = 3 \cdot 7$$

The rest of the solution is the same as in Example 2. The purpose of Example 3 is to show how to solve a proportion when the variable is in the denominator. Cross multiplication still works.