Solving One-Step Linear Equations

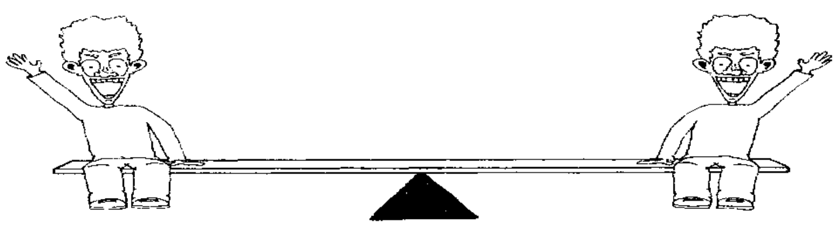
We have reviewed the basics. We are ready to learn how to solve linear equations, a basic skill for all later math contents.

Basic Principles of Solving Linear Equations

When you are solving a linear equation like $2(x+3)-4=14-x$, these are the basic principles:

- You are done when the variable is alone on one side of the equal sign, like $x=-12$ or $\frac{2}{3}=y$.
- When we solve linear equations, the key concept is to "undo", or "do the opposite". For example, to get rid of the $+2$ in $x+2$, we do the opposite of $+2$, which is -2.
- If you do something to one side of the equal sign, you must do the same to the other side. For example, for $x+2=5$, we subtract 2 on **both** sides of the equal sign, and get $x=3$.

Think of the equation as a teeter-totter on a playground. Let's say that there is a person on each side of the teeter-totter and they both weigh the same amount. Both of the two people have their feet off the ground. The teeter-totter is balancing perfectly level. If you hand a five-pound weight to one of them, the teeter-totter will no longer be balanced. To remain balanced, both of the people need to be handed the five-pound weights. Equations are just like the teeter-totter and your goal is to keep the equation balanced. Whatever you do to the left side of equal sign, you must do the same to the other side.



A Five-Step Process

You need to memorize these 5 steps when you solve a linear equation, until you can do them without too much thinking:

1. Get rid of parentheses by distributive property.
2. Combine like terms.
3. Move variable terms to one side of the equal sign.

4. Move number terms to the other side of the equal sign.
5. Get rid of the number in front of the variable.

Don't try to memorize them before you go through enough examples, so each step makes sense.

For easy equations like those in Example 1, we only use Step 5. The key word is "undo".

[**Example 1**] Solve the following equations for x.

$$x+2=10$$
$$x+2-2=10-2$$
$$x=8$$

$$x-2=10$$
$$x-2+2=10+2$$
$$x=12$$

$$2x=10$$
$$\frac{2x}{2}=\frac{10}{2}$$
$$x=5$$

$$\frac{x}{2}=10$$
$$2\cdot\frac{x}{2}=2\cdot 10$$
$$x=20$$

We can easily check these solutions by plugging the solutions into the original equations.

At this level of math, we don't write the division symbol any more. Instead, we use the fraction line. So $\frac{x}{2}$ means "*x* divided by 2". To "undo" the division in $\frac{x}{2}$, we multiplied 2 on both sides of the equal sign.

Let's summarize how to "undo" in Example 1:

- To get rid of "plus", we "minus". (Problem 1)
- To get rid of "minus", we "plus". (Problem 2)
- To get rid of "multiply", we "divide". (Problem 3)
- To get rid of "divide", we "multiply". (Problem 4)

I need to point out one very common mistake:

$$-2x=6$$
$$-2x+2=6+2$$
$$x=8$$

If we plug $x=8$ into $-2x=6$, obviously $x=8$ is not the solution. What was wrong?

Note that $-2x$ means "negative 2 times *x*", and thus to get rid of the -2 in front of x, we have to divide both sides by -2:

$$-2x=6$$
$$\frac{-2x}{-2}=\frac{6}{-2}$$
$$x=-3$$

Now the solution checks: $-2\cdot(-3)=6$

A special type of equation deserves some attention:

[**Example 2**] Solve $-x = 4$ for x.

Recall from an earlier lesson that the negative sign means "negative one times". For example, $-2 = -1 \cdot 2$. We could solve $-x = 4$ by "undo":

$$-x = 4$$
$$-1 \cdot x = 4$$
$$\frac{-1 \cdot x}{-1} = \frac{4}{-1}$$
$$x = -4$$

However, since $(-1) \cdot (-1) = 1$, we could also do:

$$-x = 4$$
$$-1 \cdot x = 4$$
$$(-1) \cdot (-1) \cdot x = (-1) \cdot 4$$
$$x = -4$$

Since multiplication is easier than division, usually we use the second method to get rid of the negative sign in front of x.

This type of equation will be used so often that if you learn to mentally multiply both sides by -1, your life would be much easier. For example:

$$-x = 2 \qquad\qquad -x = -2$$
$$x = -2 \qquad\qquad x = 2$$

Fraction Shortcut

Everyone loves fractions! :) Let's learn how to handle fractions in equations. We need to review an important pattern first. We know the fraction line means "divide". For example, $\frac{6}{2} = 3$ and $\frac{10}{5} = 2$.

There are two ways to do the problem $\frac{2}{3} \cdot 3$.

We could use the "normal" way to multiply fractions: $\frac{2}{3} \cdot 3 = \frac{2}{3} \cdot \frac{3}{1} = \frac{6}{3} = 2$.

Or, since the fraction line means "divide", we could use a shortcut: $\frac{2}{3} \cdot 3 = 3 \div 3 \cdot 2 = 1 \cdot 2 = 2$.

Let practice this shortcut a few more times:

$\frac{3}{5}\cdot 5 = 5 \div 5 \cdot 3 = 1 \cdot 3 = 3$ $\qquad$ $\frac{3}{5}\cdot 10 = 10 \div 5 \cdot 3 = 2 \cdot 3 = 6$

$\frac{3}{5}\cdot 15 = 15 \div 5 \cdot 3 = 3 \cdot 3 = 9$ $\qquad$ $\frac{7}{3}\cdot 6 = 6 \div 3 \cdot 7 = 2 \cdot 7 = 14$

This shortcut only works when the denominator goes into the integer. Otherwise, we have to use the normal way:

$$\frac{3}{5}\cdot 4 = \frac{3}{5}\cdot\frac{4}{1} = \frac{12}{5}$$

Fortunately, when we solve equations, we can choose which integer to use, implying we can use the shortcut in most cases.

Solve One-Step Equations with Fractions

[**Example 3**] Solve $\frac{1}{3}p = 2$ for p.

[**Solution**] To get rid of $\frac{1}{3}$, we will multiply both sides of the equation by 3, because $\frac{1}{3}\cdot 3 = 3 \div 3 \cdot 1 = 1$ by the pattern we learned. The solution is:

$$\frac{1}{3}p = 2$$
$$3\cdot\frac{1}{3}p = 3\cdot 2$$
$$p = 6$$

[**Example 4**] Solve $\frac{2}{3}p = 2$ for p.

[**Solution**] Again, by the shortcut, $\frac{2}{3}\cdot 3 = 3 \div 3 \cdot 2 = 2$. So we multiply both sides by 3 to get rid of $\frac{2}{3}$:

$$\frac{2}{3}p = 2$$
$$3\cdot\frac{2}{3}p = 3\cdot 2$$
$$2p = 6$$
$$\frac{2p}{2} = \frac{6}{2}$$
$$p = 3$$

Another method to solve $\frac{2}{3}p = 2$ is to recognize that $\frac{3}{2} \cdot \frac{2}{3} = 1$, so we multiply both sides of the equation by $\frac{3}{2}$, and we have:

$$\begin{aligned} \frac{2}{3}p &= 2 \\ \frac{3}{2} \cdot \frac{2}{3}p &= \frac{3}{2} \cdot 2 \\ p &= 3 \end{aligned}$$

You can decide which method to use.

[**Example 5**] Solve $-\frac{2}{3}p = 5$ for p.

[**Solution**] Method 1: We simply multiply both sides by 3 to get rid of the fraction, and then deal with the rest fraction-free.

$$\begin{aligned} -\frac{2}{3}p &= 5 \\ 3 \cdot (-\frac{2}{3})p &= 3 \cdot 5 \\ -2p &= 15 \\ \frac{-2p}{-2} &= \frac{15}{-2} \\ p &= -\frac{15}{2} \end{aligned}$$

Method 2: Notice that $(-\frac{3}{2}) \cdot (-\frac{2}{3}) = 1$, so we multiply both sides of the equal sign by $-\frac{3}{2}$:

$$\begin{aligned} -\frac{2}{3}p &= 5 \\ -\frac{3}{2} \cdot (-\frac{2}{3})p &= -\frac{3}{2} \cdot 5 \\ p &= -\frac{15}{2} \end{aligned}$$