Solve Multi-Step Linear Equations

Let's review those 5 steps to solve linear equations:

1. Get rid of parentheses by distributive property.

2. Combine like terms.

3. Move variable terms to one side of the equal sign.

4. Move number terms to the other side of the equal sign.

5. Get rid of the number in front of the variable.

Example 1 shows how to solve equations involving Step 2 through 5.

[**Example 1**] Solve $2x + 3 - 5x = 5 - x + 8$ for x.

[**Solution**] Before moving variable terms to one side (Step 3), we should combine like terms on both sides of the equation (Step 2) to make the equation simpler:

$$2x + 3 - 5x = 5 - x + 8$$
$$2x + 3 + (-5x) = 5 + (-x) + 8$$
$$-3x + 3 = 13 - x$$

The second step is optional, but it makes things clear.

Note that we can only combine like terms if they are on the same side of the equal sign. For example, we combined $2x$ and $-5x$ into $-3x$ on the left side, and combined 5 and 8 into 13 on the right side. However, we may not combine $2x$ on the left side and $-x$ on the right side.

Next, we will use Step 3, 4 and 5 to solve the equation.

$$-3x + 3 = 13 - x$$
$$-3x + 3 + 3x = 13 - x + 3x$$
$$3 = 13 + 2x$$
$$3 - 13 = 13 + 2x - 13$$
$$-10 = 2x$$
$$\frac{-10}{2} = \frac{2x}{2}$$
$$-5 = x$$

Plugging in $x = -5$ to check the solution is left as an exercise.

Example 2 shows how to solve equations with parentheses.

[**Example 2**] Solve $2(x-3) = 3+5$ for x.

[**Solution**] In Step 1, we use distributive property to get rid of parentheses:

$$2(x-3) = 3+5$$
$$2x-6 = 3+5$$

Then we follow through Step 2, 3, 4, and 5.

$$2x-6 = 3+5$$
$$2x-6 = 8$$
$$2x-6+6 = 8+6$$
$$2x = 14$$
$$\frac{2x}{2} = \frac{14}{2}$$
$$x = 7$$

Solution: $x = 7$ is the solution of $2(x-3) = 3+5$.

Be careful when distributing a negative number into the parentheses.

[**Example 3**] Solve $10-2(y-3) = 14$ for y.

[**Solution**] It helps to change "minus" to "plus negative", so it's clear we are distributing a negative number into the parentheses:

$10-2(y-3) = 14$ Next step: Change "minus" to "plus a negative".

$10+(-2)[y+(-3)] = 14$ Next step: Get rid of parentheses with distributive property.

$10+(-2)y+(-2)(-3) = 14$

$10-2y+6 = 14$ Next step: Combine like terms.

$16-2y = 14$ Next step: Move numbers to the other side of equal sign.

$16-2y-16 = 14-16$

$-2y = -2$ Next step: Get rid of the number in front of variable.

$\dfrac{-2y}{-2} = \dfrac{-2}{-2}$

$y = 1$

It's a very common mistake to change $-2(y-3)$ into $-2y-6$. Don't do that!

Be even more careful when distributing a negative sign into the parentheses.

[**Example 4**] Solve $9-(3+z)=3z-(6z-4)$ for z.

[**Solution**] Remember the negative sign means "negative one times".

$9-(3+z)=3z-(6z-4)$	Next step: Change "minus" to "plus negative".
$9+(-1)(3+z)=3z+(-1)[6z+(-4)]$	Next step: Get rid of ( ) by distributive property.
$9+(-1)3+(-1)z=3z+(-1)6z+(-1)(-4)$	
$9-3-z=3z-6z+4$	Next step: Combine like terms.
$6-z=-3z+4$	Next step: Move variables terms to one side.
$6-z+3z=-3z+4+3z$	
$6+2z=4$	Next step: Move number terms to the other side.
$6+2z-6=4-6$	
$2z=-2$	Next step: Get rid of the number in front of variable.
$\dfrac{2z}{2}=\dfrac{-2}{2}$	
$z=-1$	

Just for a note: If there is a plus sign in front of parentheses, we can actually simply get rid of the parentheses, because a value will not change if it multiplies positive one.

For example: $3+(x+1)$ can be changed to $3+x+1$.