

# Solving Inequalities

Welcome back! There are many situations where you may want a value to be smaller than a value or greater than a value. When you are calculating your bills for instance, you would want the bills to be less than your income. Similarly, if you were calculating your net income, you would want it to be greater than or equal to your expenses for the pay period. These examples illustrate the need for symbols other than an equals sign.

## *Inequality Symbols*

Today will learn inequalities, like  $x \leq 3$ . The good news is that solving an inequality follows very similar procedures (with one difference) as solving an equation, which we learned earlier. The bad news is that you have to memorize quite a few new terms and notations. Let's learn new terms first.

There are four inequality symbols. They are:

$<$	less than
$>$	greater than
$\leq$	less than or equal to
$\geq$	greater than or equal to

To recall which side is bigger, the middle school trick still works: The alligator eats the bigger number.

- For  $x > 3$ , the alligator eats  $x$ , meaning  $x$  must be bigger than 3.
- For  $x \leq 3$ , the alligator eats 3, meaning  $x$  must be smaller than or equal to 3.

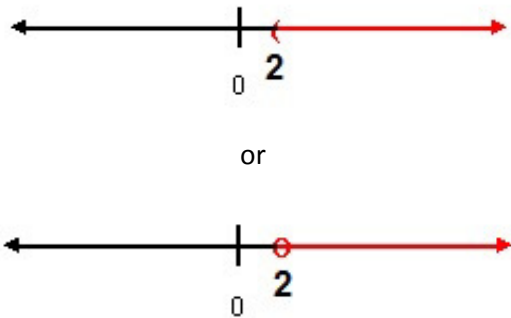
The difference between  $>$  and  $\geq$  is:

- $2 \geq 2$  is true, because 2 is equal to 2.
- $2 > 2$  is false, because 2 is not greater than 2.

## Set Builder Notation and Interval Notation

There are different ways to represent an inequality.


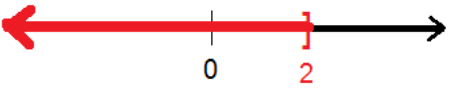
**Table 1: Different ways to represent  $x > 2$**

Inequality	$x > 2$
Graph	
Set Builder Notation	$\{x \mid x > 2\}$
Interval Notation	$(2, \infty)$

- Each notation represents the same set of numbers: all numbers greater than 2.
- Notice that a parenthesis or open circle is used in the graph, implying the number 2 is not included in the set. If the inequality is  $x \geq 2$ , a bracket [ or a closed circle would be used.
- We read  $\{x \mid x > 2\}$  this way: the set of  $x$  such that  $x$  is greater than 2. The  $\mid$  symbol is read as "such that". It's ok to write  $\{x \mid 2 < x\}$  for this problem. It is equivalent to  $\{x \mid x > 2\}$ .
- In interval notation, the "sideway 8" represents infinity. In interval notation, parentheses, ( and ), mean "exclusive"; brackets, [ and ], mean "inclusive". In this example,  $(2, \infty)$  means any number between 2 and infinity, but not including 2 and infinity. Actually, there is no such a number as infinity, so you would never see  $\infty$ ]
- In interval notation, we always write the smaller number first, and the bigger number next. It is not ok to write  $(\infty, 2)$ . This is different from set builder notation.
- In set notation, we use the symbol  $\{ \}$ , meaning "a set of number". In interval notation, we use a combination of (, [, ), ]. Note the difference!

Let's look at another example.

**Table 2: Different ways to represent  $x \leq 2$**

Inequality	$x \leq 2$
Graph	 <p>or</p> 
Set Builder Notation	$\{x \mid x \leq 2\}$
Interval Notation	$(-\infty, 2]$

- This time, in the graph, a closed circle or bracket  $]$  is used, implying the number 2 is included in the set. Compare this with the last example.
- It's ok to write  $\{x \mid x \leq 2\}$  as  $\{x \mid 2 \geq x\}$ . They are equivalent.
- In interval notation, we may not write  $[2, -\infty)$ , as the smaller number must come first.

## ***Solve Linear Inequalities***

First, let's recall the procedures to solve linear equations:

1. Simplify both sides of the equation if possible.
2. If the variable you are solving for is not on one side of the equation, then move it by adding its opposite to **both sides** of the equation.
3. Move any terms being added to or subtracted from the variable to the other side of the equation by performing the opposite operation to **both sides** of the equation. For instance, if the term is added to the variable, subtract from both sides; if the term is subtracted from the variable, add it to both sides of the equation.
4. Move any items being multiplied or divided by the variable by performing the opposite operation to **everything** in the equation. For instance, if the term is multiplied by the variable, divide both sides; if the term is divided into the variable, multiply to both sides of the equation.
5. Check your answer.

**Note: if you do something to one side of an equation, you must do it to the other side of the equation.**

If you change "equations" to "inequalities", these are the procedures to solve linear inequalities, with one exception.

Consider the inequality  $2 > 1$ . If both sides are multiplied by 1, we still have  $2 > 1$ .

However, if both sides are multiplied by  $-1$ , we have  $-2 < -1$ .

Here is the special rule about inequality: **If you multiply or divide by a negative number, you need to flip your inequality symbol over.** If you wonder why, multiply both sides of  $2 > 1$  by  $-1$ , and look at the result  $-2 < -1$ .

Here is a common confusion: If you multiply both sides of  $-2 < -1$  by 1, we still have  $-2 < -1$ . The inequality symbol didn't change direction, because we multiplied both sides by a positive number.

**[Example 1]** Solve  $2(11 + 5x) < 3(6 + 4x)$ , and write the solution in both set notation and interval notation. Finally, graph the solution set.

**[Solution]**

$$2(11 + 5x) < 3(6 + 4x)$$

First simplify by distributing.

$$22 + 10x < 18 + 12x$$

Now let's subtract  $12x$  from both sides to get the variable on the left side. (Note: unless you have a compound inequality (which we will do later), you will always want the variable on the left side because it will help you with your graphing.)

$$22 + 10x - 12x < 18 + 12x - 12x$$

$$22 - 2x < 18$$

Now we need to move the 22, so we will subtract 22 from both sides of the inequality.

$$22 - 22 - 2x < 18 - 22$$

$$-2x < -4$$

Now divide both sides by  $-2$ , and since we are dividing by a negative, we need to flip the inequality symbol over.

$$\frac{-2x}{-2} > \frac{-4}{-2}$$

$$x > 2$$

In Table 1 above, you can see this solution in set notation, interval notation, and by a graph.

Scroll down for another example.

**[Example 2]** Solve  $\frac{2}{3}x - \frac{1}{2} \leq \frac{5}{6}$ . Write your solution in set notation, interval notation and by a graph.

**[Solution]**

Everything is simplified, so let's get rid of the fractions by multiplying by the common denominator of 6. Note: we do not have to flip symbols since we are multiplying by a positive number.

$$6\left(\frac{2}{3}x\right) + 6\left(\frac{-1}{2}\right) \leq 6\left(\frac{5}{6}\right)$$

In the first term, the 3 goes into the 6 twice, in the second term the 2 goes into the 6 three times, and in the last term the 6 goes into the 6 once, to get:

$$4x - 3 \leq 5$$

Now we need to move the  $-3$ , so we will add 3 to both sides of the inequality.

$$4x - 3 + 3 \leq 5 + 3$$

$$4x \leq 8$$

Now we need to divide both sides by 4 to get the variable by itself. Since we are dividing by a positive number, we **do not** flip the symbol.

$$\frac{4x}{4} \leq \frac{8}{4}$$

$$x \leq 2$$

Table 2 above shows all different ways to represent this solution.