**Slope-Intercept Form Equation:** \( y = Mx + B \)

We will start with examples on real-life applications of linear equations.

**Example 1** In a region, this year's average temperature is 56.42 Fahrenheit degrees. The temperature has been increasing at an average rate of 0.02 Fahrenheit degrees per year. At this rate, how many years later will this region's average temperature reach 59 Fahrenheit degrees?

**Solution** Notice that the rate of change is constant—0.02 degrees per year. When the rate of change is constant, we can use a linear equation to model this situation:

\[
y = 0.02x + 56.42
\]

This linear equation is in slope-intercept form, where \( x \) represents the number of years passed since this year, and \( y \) represents the average temperature in a certain year.

For example, when we plug in \( x=0 \), we have \( y=56.42 \). This means when 0 year passed (this year), the average temperature is 56.42 degrees.

When we plug in \( x=1 \), we have \( y=0.02 \cdot 1+56.42=56.44 \). This means 1 year later (next year), the average temperature will be 56.44 degrees.

The number in front of \( x \) is the rate of change, called **slope**. The number 56.42 in this equation is \( y \)'s value when \( x=0 \). In this situation, 56.42 is the average temperature of this year, the year when \( x=0 \). If we graph this point, \((0, 56.42)\), it would be on the \( y \)-axis since the \( x \) value is 0. In other words, this point is where the line crosses the \( y \)-axis. This is why we call the ordered pair \((0, 56.42)\) the **y-intercept** of the line \( y = 0.02x + 56.42 \).

We can use this linear model to make predictions now. For example, 10 years later, the average temperature would be:

\[
y = 0.02 \cdot 10 + 56.42 = 56.62 \text{ degrees.}
\]

To find when the temperature will become 59 degrees, we plug \( y=59 \) into \( y = 0.02x + 56.42 \), and we have:

\[
59 = 0.02x + 56.42
59 - 56.42 = 0.02x + 56.42 - 56.42
2.58 = 0.02x
0.02 \cdot 2.58 = 0.02 \cdot 0.02
0.129 = x
\]

**Solution**: The average temperature in this region will reach 59 Fahrenheit degrees 129 years later.
[Example 2] An Internet Service Provider loses 2,300 dial-up customers every year. Right now it has 45,000 dial-up customers. After how many years will this provider have 10,000 dial-up customers left?

[Solution] First, let's write an equation. Since the number of customers is decreasing, the slope will be negative, -2300 people per year in this situation. This year, there are still 45,000 dial-up customers, so the y-intercept is (0,45000). The equation is:

\[ y = -2300x + 45000 \]

where \( x \) represents the number of years passed since this year, and \( y \) represents the number of dial-up customers.

To find when there will be 10,000 customers left, we plug in \( y=10000 \), and solve for \( x \):

\[
10000 = -2300x + 45000 \\
10000 - 45000 = -2300x + 45000 - 45000 \\
-35000 = -2300x \\
\frac{-35000}{-2300} = x \\
15.22 \approx x
\]

Solution: Approximately 15.22 years later, the provider will have 10,000 dial-up customers left.

In the next few examples, the slope and/or y-intercept is not easily identify.

[Example 3] Identify the slope and y-intercept of the following lines.

Line 1: \( y = -2 + 3x \)

Note that the equation is not in the slope-intercept form \( y = Mx + B \). We need to change it to the slope-intercept form first:

\[
y = -2 + 3x \\
y = 3x - 2
\]

Now we can see the slope is 3 and the y-intercept is (0, -2).
Line 2: \( y = \frac{1}{3} - \frac{2}{3}x \)

For Line 2, we again need to change the equation to slope-intercept form:

\[
y = \frac{1}{3} - \frac{2}{3}x = -\frac{2}{3}x + \frac{1}{3}
\]

The slope is \(-\frac{2}{3}\), and the \(y\)-intercept is \((0, \frac{1}{3})\).

Line 3: \( y = x + 2 \)

For Line 3, there is no number in front of \(x\), and we have to treat \(x\) as \(1\cdot x\).

The slope of Line 3 is 1, and the \(y\)-intercept is also \((0, 2)\).

Line 4: \( y = 2 - x \)

For Line 4, we need to change the equation to slope-intercept form:

\[
y = 1 - x = -x + 1
\]

There is no number but a negative sign in front of \(x\), and we need to treat \(-x\) as \(-1\cdot x\).

The slope of Line 4 is -1, and the \(y\)-intercept is \((0, 2)\).

Line 5: \( y = 2x \)

For Line 5, there is no \(y\)-intercept. We can treat the line as \(y = 2x + 0\).

The slope of Line 5 is 2, and the \(y\)-intercept is \((0, 0)\).
Line 6: \( y = 2 \) and Line 7: \( x = 2 \)

To find the slope and \( y \)-intercept of these two special lines, we need to find a pattern first. Look at these lines:

![Diagram of lines with labels: Purple line: \( y=3x \), Light blue line: \( y=2x \), Black line: \( y=x \), Red line: \( y=0.5x \), Blue line: \( y=0.25x \).]

Note that the bigger a line's slope, the "more tilted" the line is. This should make sense, because slope means "rate of change." When \( x \) value grows by 1, the purple line's \( y \) value grows by 3, while the red line's \( y \) value grows by only 0.5. That's why the purple line is "more tilted" than the red line.

Now think about these two questions:

What's a line's slope if the line is horizontal?

What's a line's slope if the line is vertical?

As a line becomes more and more "flat", its slope becomes smaller and smaller. So if a line is horizontal, its slope would be 0.

As a line becomes more and more "tilted", its slope becomes bigger and bigger. So if a line is vertical, its slope would be "infinitely large."

Once you understand this pattern, let's look at the graph of vertical and horizontal lines.
Now we can find the slope and $y$-intercept of Line 6 and Line 7.

**Line 6:** $y = 2$ and **Line 7:** $x = 2$

For the line $y = 2$, its slope is 0, and its $y$-intercept is $(0, 2)$.

For the line $x = 2$, its slope is **undefined** (because it's infinitely large), and it does not have a $y$-intercept, because the line does not cross the $y$-axis.

An alternative way of saying a slope is undefined is to say the slope **does not exist**.