Slope

What do you think of when you hear the word slope? Draw a picture of this. Often when people think of the word slope, they think of a hill or a mountain. Imagine that you are hiking up a hill. As you walk up the hill what happens to your position? Don't you move forward and at the same time you move upwards as seen in the picture.

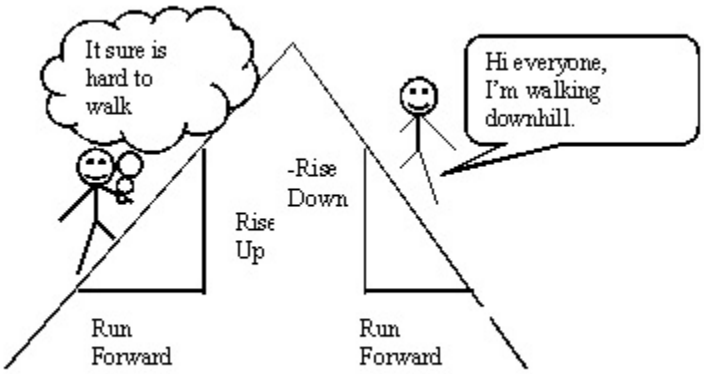

Figure 1: Understanding slope by going up and down a hill

Slope is the ratio of your rise to your run, written as: $slope = \frac{rise}{run}$. If you are going uphill, your rise is positive because you are going up and your run is positive because you are going forward, so the slope = $\frac{+rise}{+run} = +slope.$

Now going downhill you are still moving forward, which is positive, but you are dropping and going down, which is negative. Thus, lines that look like you are going downhill as you travel in the direction you read, will have a negative rise because the downhill slope = $\frac{-rise}{+run} = -slope.$

This is nice to know, because if you are graphing and your slope is positive, as you move from left to right across the paper the line should look like you are going uphill. If you graph a line that has a negative slope, as you move from left to right across the paper the line should look like you are going downhill.

The letter that will always represent slope is *m*. Now let's look at slope on a coordinate plane.

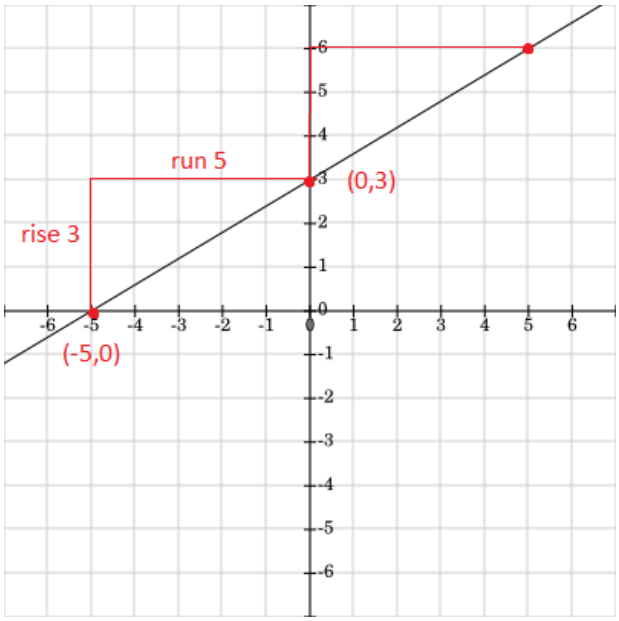

Figure 2: slope of 3/5

Let's pretend that the graph is your hill and you are walking on the surface of the hill from the point (-5, 0) to the point (0, 3) as seen above. As you walk, your movement forward on the *x*-axis from the –5 to the point (0, 0) directly below the other point, you can count that you travel 5 units forward, and since forward is positive, this is a positive 5 for the run. Your movement up from (0, 0) to the point (0, 3) is 3 units and since you are going up this is a positive 3. Since $slope = \frac{rise}{run}$, the slope of the line is $m = \frac{3}{5}$. If you plot the above points on a piece of graph paper and connect them with a ruler as I did in the picture above, you will see that the line you create can be extended to go on forever and to get from one point to the next you can just count 5 grid marks on the graph paper forward and then 3 grid marks up and you will always find another point.

Now, use some graph paper, plot (-2, 4) and (5, 1), and draw a line that goes through the points. Try to find the slope by counting the grid marks on your graph paper. It also may help you to draw a triangle for the rise over run as seen on the top of the next page.

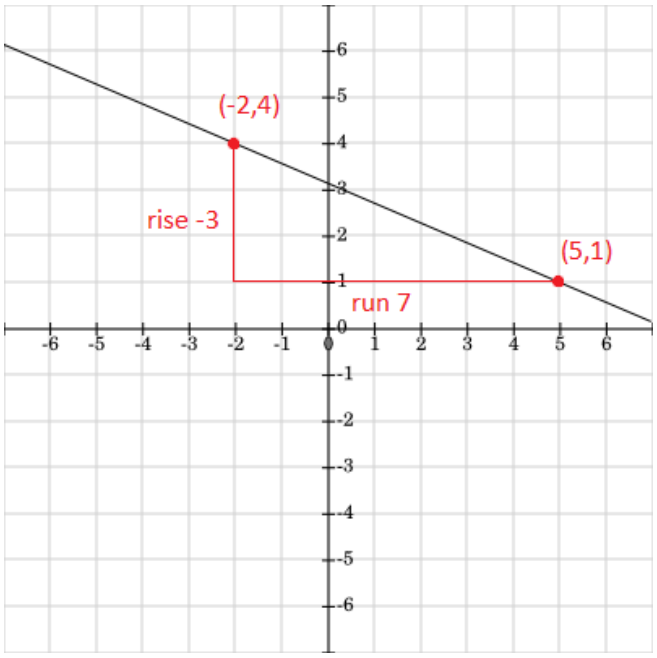

Figure 3: slope of -3/7

You should be able to count down 3 from the level of 4 down to the level of 1 on the *y*-axis. This is a –3 for a rise. You should be able to count forward 7 on the *x*-axis from –2 to 5. This gives us 7 for the run and a slope of $m = \frac{-3}{7}$.

Slope Formula

You can also find the slope without graphing by using the slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$. The 2 and 1 in the formula are subscripts that mean that the values are from two different points.

Look at the problem you just did on the graph. Let $(x_1, y_1) = (-2, 4)$ and $(x_2, y_2) = (5, 1)$. Now plugging into the formula we get:

$$m = \frac{1-4}{5-(-2)} = \frac{1-4}{5+2} = \frac{-3}{7}$$

This is what we got when you graphed the two points on the coordinate plane. You can graph to find the slope ratio or use the formula. It is up to you as to which method you want to use, unless the directions on your problem ask you to use a specific method.

It's always a good idea to add parentheses when you plug negative numbers into a formula.

Applications of Slopes

Slope can be found in many aspects of our world around us. Have you ever driven over a mountain and seen a sign that says "6% downgrade for the next 5 miles." Have you ever wondered what that really means other than the mountain is steep? Well what that means can be found by the fact that $6\% = \frac{6}{100}$ and since it is a downgrade this means the road will drop 6 feet for every 100 feet you move forward. That is a big drop! What will happen if you travel a mile? A mile is 5280 feet which means that you will drop 6 feet 52.8 times, for a total of 316.8 feet. So in one mile you will drop 316.8 feet. Wow! No wonder the mountain seems so steep!

Slope actually helped me to remodel a room in my barn. I had a wall that was shaped like the picture below. I was putting pine paneling on the wall and needed to cut the boards so they would line up with the angle of the roof. Being the math lover that I am, I figured I could use some of my skills to save time. I measured the height of both sides of the wall and the length of the wall on the floor, then I could find the slope in inches and find how much of a drop the roof makes for every 4 inches of the run, which is how wide the boards were. It ended up that the roof had a drop of ¾ inch for every 4 inches, so I was able to cut all the boards by measuring one side, then making the other side ¾ inches less, then drawing a line and cutting. With each new board I would use my last measurement from the previous board on one side, then ¾ inch less on the other side until all the boards were cut. When I put them on the wall they lined up perfectly.

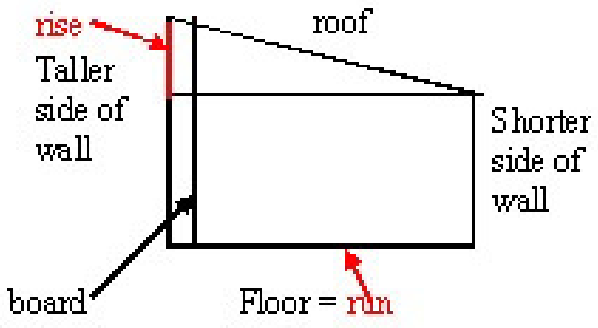

Figure 4: Use of slope when building a roof