Simplifying Expressions

We can change $4(x-2)+3(x-1)$ into $7x-11$, much simpler. We will learn how to do this.

Combine Like Terms

We can combine two terms if they are "like terms", like $x^2+x^2=2x^2$, $2x-4x=-2x$. This is a pretty easy concept.

Just remember that if there is no number in front of a variable, add 1 or -1 in front of the variable. For example, $x=1x$, and $-x=-1x$. This is because any number multiplied by 1 will not change value.

[**Example 1**] $3x-y+x-4y-10=2x-5y-10$

- For many students, it helps to first change all subtraction to "adding a negative":

  $$3x-y+x-4y-1=3x+(-y)+x+(-4y)+(-10)$$

- A common mistake is to do $3x-x$, because a minus sign follows $3x$. Once we change all subtraction to "adding a negative", we can clearly see the minus sign behind $3x$ actually belongs to the $-y$ term.
- Note that $3x+x=4x$, and $-y-4y=-5y$, both due to the invisible 1.
- Finally, the number -10 is left alone because no other numbers can be combined with it.

[**Example 2**] $2x^2-2xy-4x+2xy-x=2x^2-5x$

- Note that $2x^2$ and $-5x$ cannot be combined because they are not like terms. The square makes a difference.
- Because $-2xy+2xy=0$, no xy terms are left.

Distributive Property

By the distributive property, $2(x+3)=2x+6$. Why?

There are two ways to explain the distributive property.

Assume you have a bag with a 3-lb object and a 4-lb object in it. Together, they weigh $(3+4)$ lb. If we double the bag's weight, we would do $2(3+4)$. This is equivalent to doubling the 3-lb object first, and then doubling the 4-lb object, or, $2\cdot3+2\cdot4$. This is why $2(3+4)=2\cdot3+2\cdot4$.

A second way to explain the distributive property is to use an area model. Assume you have a house with two rooms.

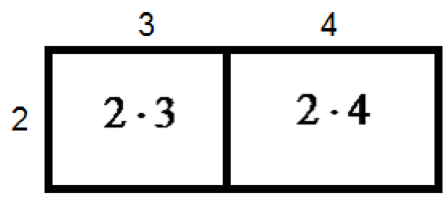

Figure 1: a house with two rooms

The first way to find the whole house's area is to do $2(3+4)$.

Another way is to add up those two rooms' area: $2\cdot 3+2\cdot 4$.

This is the second way to understand $2(3+4)=2\cdot 3+2\cdot 4$.

[**Example 3**] $2(x-y)=2x-2y$

[**Example 4**] $(x-y)=x-y$

If there is no number in front of the parentheses, you could imagine a 1 in front, and distribute in 1. Since any number multiplied by 1 does not change value, $(x-y)=x-y$. The lesson is that if there is no number in front of parentheses, we can safely ignore the parentheses.

[**Example 5**] $-(x-y)=-1[x+(-y)]=-1\cdot x+(-1)\cdot(-y)=-x+y$

Such a simple problem needs a lot of explanation.

- First, we may not ignore the negative sign outside the parentheses. We must treat is as -1.
- Second, before we distribute -1 into the parentheses, we change subtraction inside the parentheses into addition. This would avoid confusion later.
- Third, notice that once I changed $x-y$ to $x+(-y)$, the outside parentheses became [ ]. This is because $[x+(-y)]$ is more clear than $(x+(-y))$ in that we can differentiate those two pairs of parentheses.
- Finally, notice that once the negative sign is distributed into the parentheses, x became $-x$, and $-y$ became y. Basically all terms changed sign. This pattern will save you a lot of time. For example, you can do this problem in one step: $-(2x+xy-3y)=-2x-xy+3y$. It's ok to use shortcut as long as you understand why.

Now let's put together distributive property and combining like terms.

[**Example 6**]

$$\begin{aligned}
&5(2x-3)-4(6x+5)\\
&=5[2x+(-3)]+(-4)(6x+5)\\
&=10x-15-24x-20\\
&=-14x-35
\end{aligned}$$

Notice that I changed $-4(6x+5)$ into $+(-4)(6x+5)$. This way, it's clear that I will distribute the number -4 into the parentheses, not 4. It is a very common mistake to simply distribute 4 into the parentheses.

Differentiate between these two facts: $x+x=2x$, but $x \cdot x = x^2$.

[**Example 7**]

$$\begin{aligned}
&t(-1-t)-(t-3t^2)\\
&=t[-1+(-t)]+(-1)[t+(-3t^2)]\\
&=t\cdot(-1)+t\cdot(-t)+(-1)\cdot t+(-1)(-3t^2)\\
&=-t-t^2-t+3t^2\\
&=2t^2-2t
\end{aligned}$$

If you are new to this subject, it helps to change subtraction to "adding a negative". After a while, you can go direction from $t(-1-t)-(t-3t^2)$ to $-t-t^2-t+3t^2$. Just keep telling yourself: Negative times negative is positive.