"Rate and Starting Value" Word Problems

This type of word problem is very important in that it is directly related to a line's equation, which you will learn later. A line's equation looks like $y = Mx + B$, where *M* is the rate of change (slope), and *B* is the starting value (*y*-intercept).

[**Example 1**] A furniture salesman earns a monthly base salary of $1,500. On top of it, he earns $80 commission for each piece of furniture he sells. Last month, he made a total of $2,220 in salary and commission. How many pieces of furniture did he sell?

[**Solution**] First, let's look at a pattern:

Number of pieces of furniture sold	Amount of commission (in dollars)
0	$80 \cdot 0 = 0$
1	$80 \cdot 1 = 80$
2	$80 \cdot 2 = 160$
3	$80 \cdot 3 = 240$
...	...
x	$80 \cdot x = 80x$

The last row shows the pattern. Since the salesman earns $80 commission for each piece of furniture sold, he would make $80x$ dollars of commission for *x* pieces of furniture sold. We call $80 in this problem the "rate of change". In a linear equation, "rate of change" is also called "slope."

His monthly income is the sum of his base salary plus his commission, which is $80x + 1500$ dollars. We call $1500 in this problem the "starting value." In a linear equation, "starting value" is called "*y*-intercept".

Last month he made $2,220 in salary and commission, so we can write an equation and then solve it:

$$80x + 1500 = 2220$$
$$80x + 1500 - 1500 = 2220 - 1500$$
$$80x = 720$$
$$\frac{80x}{80} = \frac{720}{80}$$
$$x = 9$$

Solution: The salesman sold 9 pieces of furniture last month.

The equation $y = 80x + 1500$ is very useful. For example, if the salesman sells 30 pieces of furniture in a month, we can do:

$$y = 80x + 1500$$
$$= 80 \cdot 30 + 1500$$
$$= 2400 + 1500$$
$$= 3900$$

to figure out that his income in that month is $3,900.

In another situation, assume in a certain month the salesman plans to make a total of $4,460. How many pieces of furniture does he need to sell? We can plug in $y = 4460$ and then solve for *x*:

$$80x + 1500 = 4460$$
$$80x + 1500 - 1500 = 4460 - 1500$$
$$80x = 2960$$
$$\frac{80x}{80} = \frac{2960}{80}$$
$$x = 37$$

This result shows he needs to sell 37 pieces of furniture to make $4,460 in a given month.

It would be very useful if you understand this formula in the context of Example 1:

$$y = Mx + B$$

where *M* means the rate of change ($80 per piece), *B* means the starting value ($1500 base pay), and *y*'s value (total monthly income) changes based on *x*'s value (number of pieces of furniture sold).

[**Example 2**] A company made a budget of $44,300 for TV advertisement. Each week, it spent $2,300. After some weeks, $9,800 were still left unspent. How many weeks have passed?

[**Solution**] Let's use the formula $y = Mx + B$.

In this problem, the rate of change is $2,300, except it is negative because the company is spending money each week, not making money. So $M = -2300$.

The company's original budget was $44,300, which is the starting value, or *B*.

Now we have the equation:

$$y = -2300x + 44300$$

where *x* is the number of weeks passed, and *y* is the money left.

The week in question has $9,800 left in the budget, so we plug $y = 9800$ into $y = -2300x + 44300$, and we have:

$$9800 = -2300x + 44300$$
$$9800 - 44300 = -2300 + 44300 - 44300$$
$$-34500 = -2300x$$
$$\frac{-34500}{-2300} = \frac{-2300x}{-2300}$$
$$15 = x$$

Solution: When there was $9,800 left in the budget, 15 weeks have passed since the budget was spent.

The equation $y = Mx + B$ applies to many other situations. Here are a few more examples:

[**Example 3**] A cell phone company charges $20 flat fee, plus $0.05 per minute of talk time. The corresponding equation is $y = 0.05x + 20$, where *x* is the number of minutes in talk time, and *y* is the monthly bill in dollars.

[**Example 4**] A company owed 1.2 million dollars when it was created. Each year, it pays back 0.2 million dollars. The corresponding equation is $y = 0.2x - 1.2$, where *x* is the number of years passed, and *y* is the amount of money the company still owes in millions of dollars.

[**Example 5**] In a certain region, the yearly average temperature decreases by 0.03 $^\circ$C. This year, the average temperature is 28.9 $^\circ$C. The corresponding equation is $y = -0.03x + 28.9$, where *x* is the number of years passed, and *y* is this region's yearly average temperature.

You can see why the linear equation $y = Mx + B$ is widely used in real life.