

Proportion Word Problems

Proportion is regularly used in real life. Let's start with a cooking problem.

[Example 1] According to a recipe, three and a half cups of orange juice are needed to make 5 gallons of punch. By this recipe, how many cups of coke are needed to make 7 gallons of punch?

[Solution] Assume we need x cups of coke to make 7 gallons of punch.

The key to set up a proportion correctly is to fill in those 4 spots in a proportion by concepts and units, instead of directly using numbers. In this example, we are dealing with the relationship between orange juice and punch. First, we write:

$$\frac{\text{orange juice in cups}}{\text{punch in gallons}} = \frac{\text{orange juice in cups}}{\text{punch in gallons}}$$

Since 3.5 cups of orange juice correspond to 5 gallons of punch, we fill the left-side fraction with these data:

$$\frac{3.5 \text{ cups of orange juice}}{5 \text{ gallons of punch}} = \frac{\text{orange juice in cups}}{\text{punch in gallons}}$$

Next, we need to put "x cups of coke and 7 gallons of punch" into the right-side fraction in this proportion. Without the concepts and units, it would be difficult to decide which number goes to where. With the concepts and units, it's easy to tell x should be in the numerator, and 7 should be in the denominator. We have:

$$\frac{3.5 \text{ cups of orange juice}}{5 \text{ gallons of punch}} = \frac{x \text{ cups of orange juice}}{7 \text{ gallons of punch}}$$

Now we can get rid of words and solve for x :

$$\begin{aligned}\frac{3.5}{5} &= \frac{x}{7} \\ 5x &= 3.5 \cdot 7 \\ 5x &= 24.5 \\ \frac{15x}{5} &= \frac{24.5}{5} \\ x &= 4.9\end{aligned}$$

Solution: By this recipe, 4.9 cups of coke are needed to make 7 gallons of punch.

If you are new to proportion word problems, make sure to write down concepts and words on scratch paper like in Example 1. One you can sort out concepts and words in your head, you can directly write proportions.

[Example 2] In a town, the owner of a \$300,000 house pays \$1,400 of property tax each year. By this rate, the owner of a \$450,000 house should pay how much property tax per year?

[Solution] Assume the owner of a \$450,000 house should pay x dollars in property tax per year.

Note that writing a proportion like this won't help:

$$\frac{\text{dollars}}{\text{dollars}} = \frac{\text{dollars}}{\text{dollars}}$$

We must add in concepts:

$$\frac{\text{value of house in dollars}}{\text{property tax in dollars}} = \frac{\text{value of house in dollars}}{\text{property tax in dollars}}$$

It's ok to write "property tax in dollars" in the numerator and "value of house in dollars" in the denominator. The answer would be the same.

Since \$300,000 in house value corresponds to \$1,400 in property tax, we have:

$$\frac{300000}{1400} = \frac{\text{value of house in dollars}}{\text{property tax in dollars}}$$

Next, since \$450,000 in house value corresponds to x dollars in property tax, we have:

$$\frac{300000}{1400} = \frac{450000}{x}$$

Now we can solve this proportion:

$$\begin{aligned} \frac{300000}{1400} &= \frac{450000}{x} \\ 300000x &= 1400 \cdot 450000 \\ 300000x &= 630000000 \\ \frac{300000x}{300000} &= \frac{630000000}{300000} \\ x &= 2100 \end{aligned}$$

Solution: the owner of a \$450,000 house should pay \$2,100 dollars in property tax per year.

Sometimes it's difficult to organize concepts, like in the next example.

[Example 3] A biologist wants to find out the number of fish living in a lake. She caught 300 fish, tagged each and then released them back into the lake. After a week, she caught 200 fish, and found that 29 were tagged. Estimate the number of fish population in this lake.

[Solution] In real life, this is what biologists actually do to estimate animal population.

Assume x fish live in this lake.

It helps to sort information this way: In the lake, 300 fish are tagged out of x fish living, while 29 fish are tagged out of 200 fish caught. The proportion looks like:

$$\frac{\text{tagged fish among caught fish}}{\text{number of caught fish}} = \frac{\text{tagged fish in the whole lake}}{\text{number of fish living in the whole lake}}$$

Fill in numbers, we have:

$$\frac{29 \text{ tagged fish among caught fish}}{200 \text{ fish caught}} = \frac{300 \text{ tagged fish in the whole lake}}{x \text{ fish living in the whole lake}}$$

Now we can solve the proportion:

$$\begin{aligned}\frac{29}{200} &= \frac{300}{x} \\ 29x &= 200 \cdot 300 \\ 29x &= 60000 \\ \frac{29x}{29} &= \frac{60000}{29} \\ x &\approx 2068.97\end{aligned}$$

Solution: There are approximately 2069 fish living in this lake.

Don't say: "There are 2068.97 fish living in this lake." :)