Parallel and Perpendicular Lines

Let's graph these two lines: $y = 2x + 3$ and $y = 2x - 1$

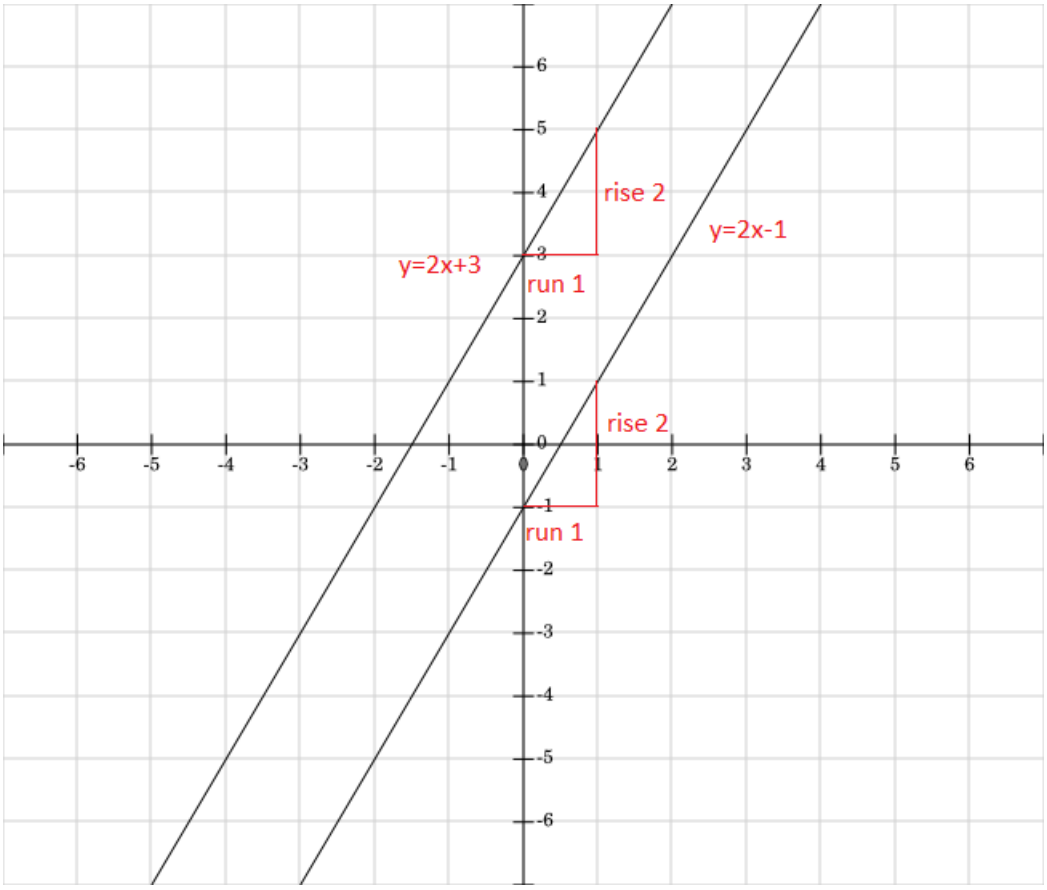

Figure 1: Graph of y=2x+3 and y=2x-1

Note that these two lines are parallel, because they are increasing at the same rate of change—each time it runs by 1 unit, it rises by 2 units.

Remember this important pattern: If two lines have the same slope, they are parallel.

[**Example 1**] Line *m*'s equation is $y = -2x + 9$. Line *n* is parallel to Line *m*, and Line *n* passes the point (2, 3). Find Line *n*'s equation.

[**Solution**] Line *n*'s equation looks like $y = Mx + B$. Since Line *m* and Line *n* are parallel, they have the same slope. So Line *n*'s equation must be $y = -2x + B$.

Next, we will find *B*'s value by plugging in the given point (2, 3) into $y = -2x + B$:

$$\begin{aligned} y &= -2x + B \\ 3 &= -2 \cdot 2 + B \\ 3 &= -4 + B \\ 3 + 4 &= -4 + B + 4 \\ 7 &= B \end{aligned}$$

Solution: Line *n*'s equation is $y = -2x + 7$.

There is another pattern when two lines are perpendicular. Look at these two pairs of lines:

$$y = 2x + 3 \text{ and } y = -\frac{1}{2}x + 3$$

$$y = \frac{2}{3}x - 4 \text{ and } y = -\frac{3}{2}x - 4$$

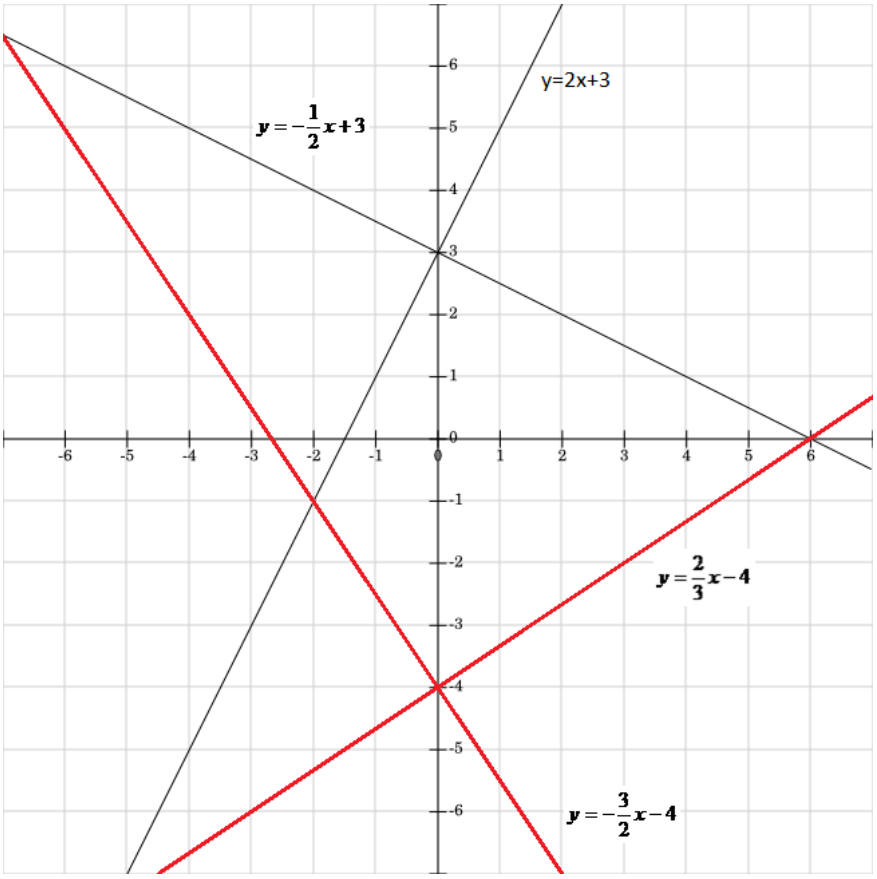

Figure 2: two pairs of perpendicular lines

When two lines are perpendicular, the product of their slopes is -1.

For $y = 2x + 3$ and $y = -\frac{1}{2}x + 3$, we have $2 \cdot (-\frac{1}{2}) = -1$;

for $y = \frac{2}{3}x - 4$ and $y = -\frac{3}{2}x - 4$, we have $\frac{2}{3} \cdot (-\frac{3}{2}) = -1$.

[**Example 2**] Line *m*'s equation is $y = -2x + 9$. Line *n* is **perpendicular** to Line *m*, and Line *n* passes the point (2, 3). Find Line *n*'s equation.

[**Solution**] Let Line *n*'s equation be $y = Mx + B$.

Since these two lines are perpendicular, the product of their slopes is -1. We have:

$$\begin{aligned} -2M &= -1 \\ \frac{-2M}{-2} &= \frac{-1}{-2} \\ M &= \frac{1}{2} \end{aligned}$$

Line *n*'s equation must be $y = \frac{1}{2}x + B$.

Next, we plug the point (2, 3) into $y = \frac{1}{2}x + B$, and we have:

$$\begin{aligned} y &= \frac{1}{2}x + B \\ 3 &= \frac{1}{2} \cdot 2 + B \\ 3 &= 1 + B \\ 2 &= B \end{aligned}$$

Solution: Line *n*'s solution is $y = \frac{1}{2}x + 2$.

Note that a line's y-intercept value does not affect a line's slope. $y = -2x + 999$ and $y = \frac{1}{2}x - 999$ are still perpendicular to each other, although they are far away from each other.