Order of Operations

Probably you remember Please Excuse My Dear Aunt Sally. Well, it's a little bit outdated.

New Way to Write PEMDAS

The new way to write it is:

Please	parentheses and absolute value: ( ) and | |
Excuse	exponent: like 2^3
My Dear	multiplication and division
Aunt Sally	addition and subtraction

First, notice that absolute value has the same priority as parentheses.

Next, notice that multiplication and division are at the same level. They cannot override each other. First come, first serve. If division comes before multiplication, we need to do division first. The same rule works for addition and subtraction. See the following examples:

$$\begin{aligned} & 3+2-1 \\ &= 5-1 \\ &= 4 \end{aligned} \qquad \begin{aligned} & 3-2+1 \\ &= 1+1 \\ &= 2 \end{aligned} \qquad \begin{aligned} & 12\cdot 3\div 4 \\ &= 36\div 4 \\ &= 9 \end{aligned} \qquad \begin{aligned} & 12\div 4\cdot 3 \\ &= 3\cdot 3 \\ &= 9 \end{aligned}$$

Of course, multiplication/division still override addition/subtraction:

$$\begin{aligned} & 1-2\cdot 3 \\ &= 1-6 \\ &= -5 \end{aligned}$$

Remember that, at this level of math, we don't use the multiplication symbol $\times$ any more. Instead, we use a dot, or use nothing whenever possible. For example, $2x$ means $2\cdot x$, $2(x+1)$ means $2\cdot(x+1)$. However, we may not omit the dot in $2\cdot 3$. Otherwise the number becomes 23.

Carefully go through each of the following examples.

[**Example 1**]

$$\begin{aligned} & 10-2(5+3) \\ &= 10-2\cdot 8 \\ &= 10-16 \\ &= -6 \end{aligned}$$

In the first step, notice that $(5+3)$ became 8. The parentheses disappeared because the only number left inside was 8. It's ok to write $10-2(8)$, though. The answer would be the same. It's not ok to omit the parentheses if the inside has a negative number! See Example 2.

In the first step, we cannot do $10-2$ first. This is because there is an invisible multiplication symbol between 2 and $(5+3)$. We have to do multiplication before subtraction.

[**Example 2**]

$$\begin{aligned} & 10-2(3-5) \\ =\ & 10-2(-2) \\ =\ & 10-(-4) \\ =\ & 10+4 \\ =\ & 14 \end{aligned}$$

This time, since -2 is left inside the parentheses, we may not omit the parentheses and write $10-2-2$. Otherwise, we would change multiplication into subtraction. We could write $10-2\cdot -2$, but this is not as nice as $10-2(-2)$. The rule is that you can only omit the parentheses if the only number inside is a positive number. The reason behind this rule is: We write as little as possible for the sake of simplicity, but not at the cost of confusion.

[**Example 3**]

$$\begin{aligned} & 2-(18-4^2)^3 \\ =\ & 2-(18-16)^3 \\ =\ & 2-2^3 \\ =\ & 2-8 \\ =\ & -6 \end{aligned}$$

We have to do what are inside the parentheses first. Inside the parentheses, we do exponent 4^2 first.

Once $(18-16)^3$ becomes $(2)^3$, we can ignore the parentheses.

Notice that $2^3=8$, not 6!

[**Example 4**]

$$\begin{aligned} & 10-2|3-5| \\ =\ & 10-2|-2| \\ =\ & 10-2\cdot 2 \\ =\ & 10-4 \\ =\ & 6 \end{aligned}$$

Absolute value symbol has the same priority as parentheses, so we have to deal with the absolute value symbol first. Recall that the absolute value symbol changes -2 into 2.

Again, in the first step, we may not do $10-2$ first, because there is an invisible multiplication symbol between 2 and $|3-5|$.

$(-2)^2$ and -2^2

This topic will be heavily tested. Make sure you understand the difference between

$$-2^2=-4 \text{ and } (-2)^2=4$$

It's easier to understand $(-2)^2=4$. Because parentheses override exponent, we cannot ignore the parentheses and do 2^2 first. We have to do $(-2)^2=(-2)(-2)=4$.

To understand $-2^2=-4$, we need to understand what a negative sign really means. Let's look at a pattern:

$$\begin{aligned} -2 &= -1\cdot 2 \\ -3 &= -1\cdot 3 \\ -4 &= -1\cdot 4 \\ &\cdots \\ -x &= -1\cdot x \end{aligned}$$

We can see the negative sign really means "negative one times a number".

So, we can treat -2^2 as $-1\cdot 2^2$. Next, we need to do exponent before multiplication, so we have:

$$-2^2=-1\cdot 2^2=-1\cdot 4=-4$$

Basically, order of operations decided $-2^2=-4$ and $(-2)^2=4$. In $-2^2=-4$, exponent overrides multiplication; in $(-2)^2=4$, parentheses override exponent.

Make sure each problem in Example 5 makes sense.

[**Example 5**]

$$\begin{array}{lllll} 5+(-2)^2 & 5-(-2)^2 & 5-2^2 & (-2)^2-1 & -2^2-1 \\ =5+4 & =5-4 & =5-4 & =4-1 & =-4-1 \\ =9 & =1 & =1 & =3 & =-5 \end{array}$$

$(-2)^3$ and -2^3

Be very careful here: $-2^3=-8$ and $(-2)^3=-8$.

This is because:

$$-2^3=-1\cdot 2^3=-1\cdot 2\cdot 2\cdot 2=-8 \qquad \text{and} \qquad (-2)^3=(-2)(-2)(-2)=-8$$

[**Example 6**]

$$\begin{array}{llll} 5-(-2)^3 & 5-2^3 & (-2)^3+8 & -2^3+8 \\ =5-(-8) & =5-8 & =-8+8 & =-8+8 \\ =13 & =-3 & =0 & =0 \end{array}$$

Complicated Order of Operations Problems

In complicated order of operation problems, it's critical to do one step a time, and copy down all the other operations as you go.

[**Example 7**]

$$\begin{aligned} & 1+3(6-2^3-2)^2 \\ =\ & 1+3(6-8-2)^2 \\ =\ & 1+3(-2-2)^2 \\ =\ & 1+3\cdot(-4)^2 \\ =\ & 1+3\cdot 16 \\ =\ & 1+48 \\ =\ & 49 \end{aligned}$$

Notice that only one operation was done in each step, and everything else was copied down with no change.