Linear Equation Applications

Today we study how to apply linear equations in some real-life scenarios.

[**Example 1**] A door-to-door vacuum salesman makes $1,000 base salary per month, plus $50 commission for each vacuum sold.

a) Write a linear equation to model the salesman's monthly income based on the number of vacuums he sells.

b) To make $2,500 total income in a certain month, how many vacuums does he have to sell?

c) Interpret the meaning of this line's slope in this scenario.

d) Interpret the meaning of this line's *y*-intercept (also called vertical intercept) in this scenario.

e) Interpret the meaning of this line's *x*-intercept (also called horizontal intercept) in this scenario.

[**Solution**]

Part a) Assume the salesman sold *x* vacuums in a certain month, which will earn him $50x$ dollars in commission. With the $1,000 base pay, his monthly income can be modeled by this equation:

$$y = 50x + 1000$$

where *y* is his monthly income in dollars, and *x* is the number of vacuums he sold.

Part b) If his total income is $2,500 in a certain month, we can plug in $y = 2500$ and solve for *x*:

$$\begin{aligned} y &= 50x + 1000 \\ 2500 &= 50x + 1000 \\ 2500 - 1000 &= 50x + 1000 - 1000 \\ 1500 &= 50x \\ \frac{1500}{50} &= \frac{50x}{50} \\ 30 &= x \end{aligned}$$

He has to sell 30 vacuums to make $2,500 in a certain month.

Part c) This line's slope is 50. It implies the salesman makes $50 for each vacuum he sells. When you interpret slope of a line, always use units, and always use the key word "every", "each", "per", etc.

Part d) To find this line's *y*-intercept, plug in $x = 0$, and we have:

$$y = 50x + 1000 = 50 \cdot 0 + 1000 = 1000$$

This line's *y*-intercept is $(0{,}1000)$. This point means if he sells no vacuum in a certain month, his total income would be $1,000.

Part e) To find this line's *x*-intercept, plug in $y = 0$, and we have:

$$\begin{aligned} y &= 50x + 1000 \\ 0 &= 50x + 1000 \\ 0 - 1000 &= 50x + 1000 - 1000 \\ -1000 &= 50x \\ \frac{-1000}{50} &= \frac{50x}{50} \\ -200 &= x \end{aligned}$$

This line's *x*-intercept is $(-200{,}0)$. This point means if he sells -200 vacuums in a certain month, his total income would be $0. This point doesn't apply in this situation. In most math models, we only look at a certain part of the data.

The line's graph should help you understand these solutions:

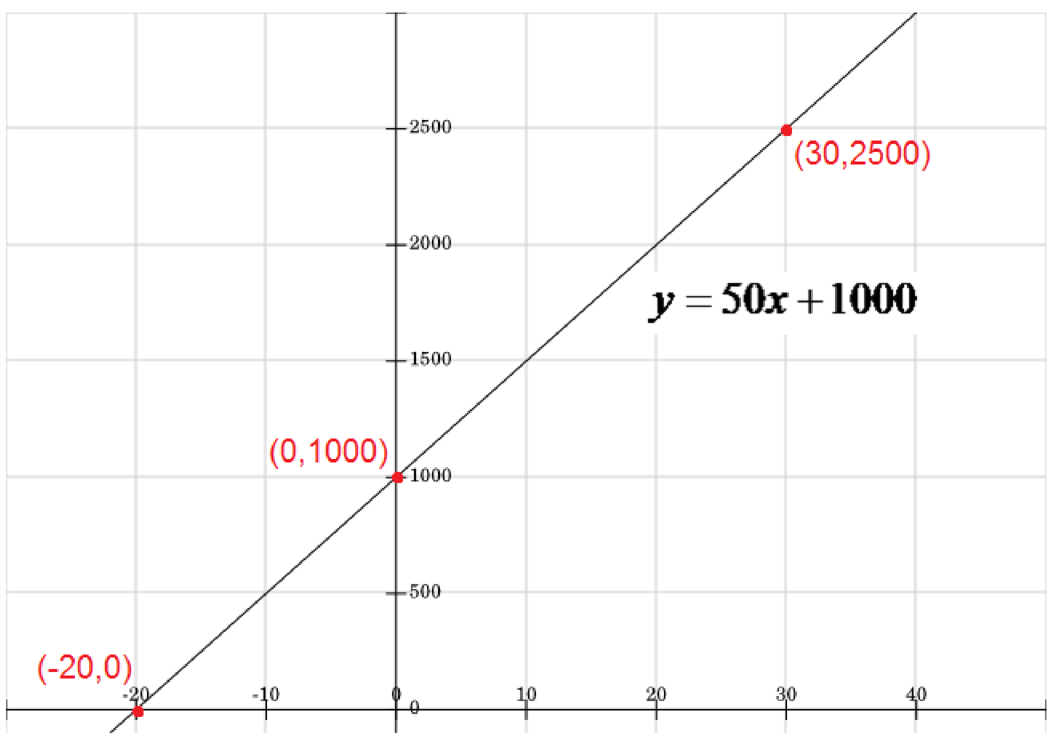

Figure 1: Example 1's solutions in graph

Next, let's look at a scenario where the *x*-intercept does make sense.

[**Example 2**] A school district has 2.3 million dollars in reserve fund. Due to the state's budget cut, the district uses 0.42 million dollars from the reserve fund every year.

a) Write a linear equation to model this school district's reserve fund as years pass.

b) Interpret the meaning of this line's slope in this scenario.

c) Interpret the meaning of this line's *y*-intercept (also called vertical intercept) in this scenario.

d) Interpret the meaning of this line's *x*-intercept (also called horizontal intercept) in this scenario.

[**Solution**]

Part a) Since the school district spends 0.42 million dollars per year, in *x* years, it will spend $0.42x$ million dollars. The district started with 2.3 million dollars in reserve fund, so the equation to model the reserve fund is:

$$y = -0.42x + 2.3$$

where *y* represents the amount of money in the reserve fund in million dollars, and *x* represents the number of years passed.

Part b) The line's slope is -0.42, implying the school district spends 0.42 million dollars every year. Compare this with Example 1, where the slope is positive because the salesman makes $50 for each vacuum sold. In this example, the school district is spending money, which is why the slope is negative.

Part c) To find this line's *y*-intercept, plug in $x = 0$, and we have:

$$y = -0.42x + 2.3 = -0.42 \cdot 0 + 2.3 = 2.3$$

This line's *y*-intercept is $(0,2.3)$. This point means, when the spending started ($x = 0$), the school district has 2.3 million dollars in reserve fund.

Part d) To find this line's *x*-intercept, plug in $y = 0$, and we have:

$$\begin{aligned} y &= -0.42x + 2.3 \\ 0 &= -0.42x + 2.3 \\ 0 - 2.3 &= -0.42x + 2.3 - 2.3 \\ -2.3 &= -0.42x \\ \frac{-2.3}{-0.42} &= \frac{-0.42x}{-0.42} \\ 5.48 &\approx x \end{aligned}$$

This line's *x*-intercept is $(5.48,0)$. This point means, 5.48 years later, the reserve fund will have $0 left. Or, in a more meaningful way, the spending must be adjusted after 5 years.

The line's graph should help you understand these solutions:

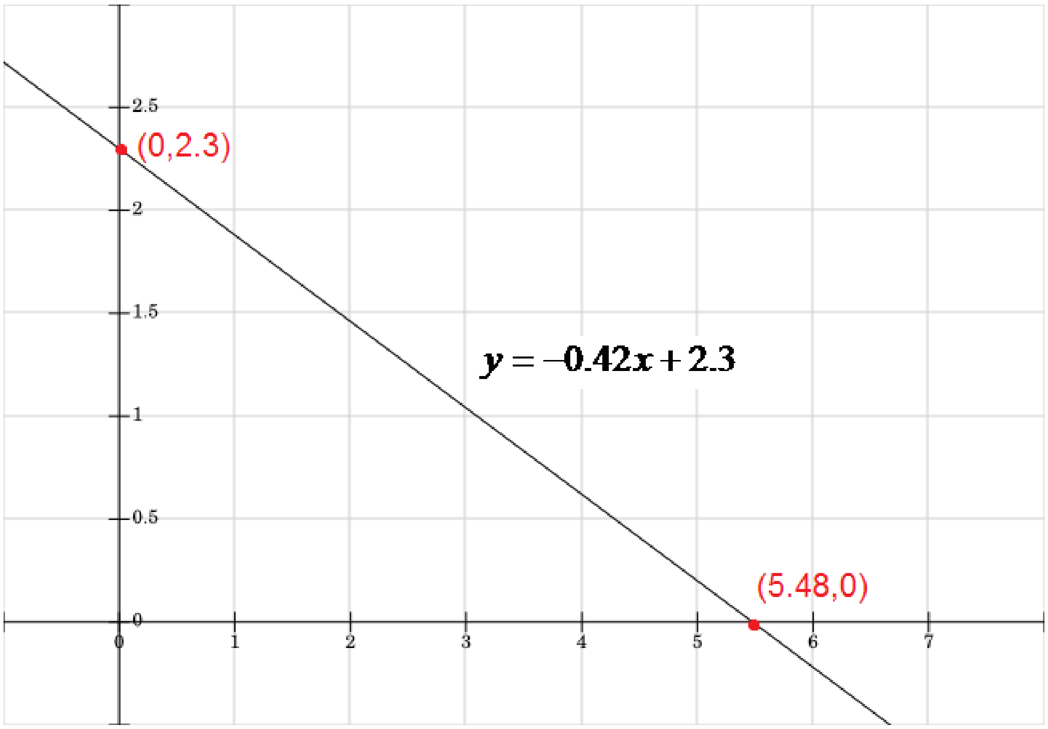

Figure 2: Example 2's solutions in graph