

Inequality Applications

There is no need to study inequalities if we cannot use them in real life. Actually, inequalities are used much more often than equations. For example, once you get a budget, you can spend any amount of money smaller than or equal to the budget. When a factory orders a type of machine part, the error must be smaller than or equal to a certain number.

[Example 1] Peter is trying to qualify for the final of a 100-meter dash sports meet. He has to run 3 times, and he will qualify for the final if his average time is smaller than 13.00 seconds. His time for the first two tries were 13.51 and 12.87 seconds. To qualify for the final, how fast does he have to run in the third try?

[Solution] Assume Peter will run x seconds in his third try. By the given condition that his average time must be smaller than 13.00 seconds, we can write and solve an inequality:

$$\begin{aligned}\frac{13.51 + 12.87 + x}{3} &< 13 \\ 3 \cdot \frac{13.51 + 12.87 + x}{3} &< 3 \cdot 13 \\ 13.51 + 12.87 + x &< 39 \\ 26.38 + x &< 39 \\ 26.38 + x - 26.38 &< 39 - 26.38 \\ x &< 12.62\end{aligned}$$

Solution: In his third try, Peter must run faster than 12.62 seconds to qualify for the final.

Note that we used $<$, instead of \leq , in the inequality, because the problem said "smaller than 13.00 seconds". If the problem said "smaller than or equal to 13.00", we would have to use \leq .

Scroll down for another example.

[Example 2] You pay your cell phone company \$30 of monthly flat fee, plus \$0.04 per minute of talk time. Next month, your cell phone budget is \$65.00. What's the maximum number of minutes you can talk over the phone next month?

[Solution] Assume you will talk x minutes over the phone next month. The total bill would be $0.04x + 30$. To keep the bill at or below \$65.00, we can write and solve an inequality:

$$\begin{aligned}0.04x + 30 &\leq 65 \\0.04x + 30 - 30 &\leq 65 - 30 \\0.04x &\leq 35 \\\frac{0.04x}{0.04} &\leq \frac{35}{0.04} \\x &\leq 875\end{aligned}$$

Solution: You can talk a maximum of 875 minutes over the phone and still stay within the budget of \$65.00.

Note that we used \leq , because it's ok to spend exactly \$65.00.