Area of Common Shapes

Area of Rectangle

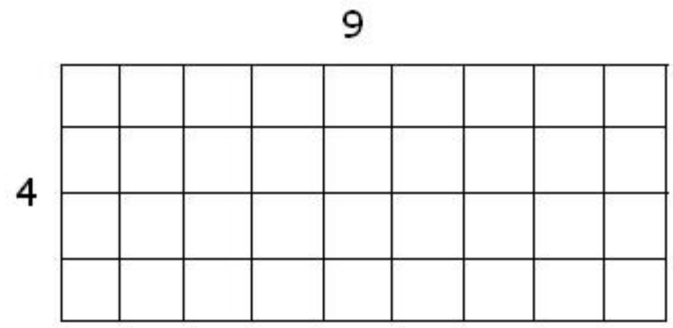

Figure 1: A 4-by-9 rectangle with 36 unit squares inside

A shape's area means the number of 1-by-1 square unit the shape covers. In Figure 1, this rectangle's area is $9 \cdot 4 = 36$ square units, as the rectangle covers 36 1-by-1 unit squares. It's fairly easy to understand that a rectangle's area formula is:

rectangle area = base · height (or length · width)

Area of triangle

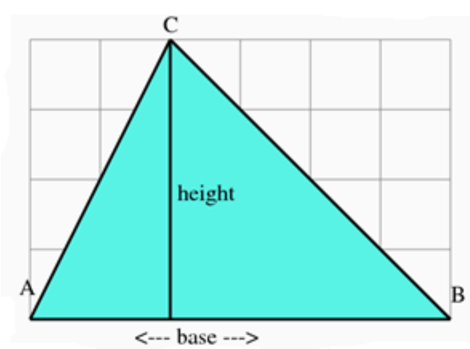

Figure 2: A triangle's area is half as big as a rectangle with the same base and height

By this graph, it's fairly easy to see why a triangle's area is half as big as the area of a rectangle with the same base and same height. Thus a triangle's area formula is:

$$\text{triangle area} = \frac{1}{2}(\text{base})(\text{height})$$

[**Example 1**] A triangle's base is 4 meters, and its height is 2.5 meters. Find its area.

[**Solution**] By the triangle area formula, the area is:

$$A = \frac{1}{2}(base)(height) = \frac{1}{2} \cdot 4 \cdot 2.5 = 5 \text{ square meters}$$

Notice that the unit of area is different from the unit of perimeter. If a rectangle's perimeter is some meters, then its area must be some **square** meters.

For simplicity, we can also write "5 square meters" as "5 m^2". The letter "m" represents "meters". Similarly, "cm" represents centimeters, "in" represents inches, etc.

Notice that "5 m^2" is different from "5^2 m":

- "5 m^2" means an area of 5 square meters. The square has nothing to do with 5.
- "5^2 m" means a length of 25 meters. The square has nothing to do with "m".

A right triangle's height is actually one of its legs, and an obtuse triangle's height lies outside the triangle. See the following figures:

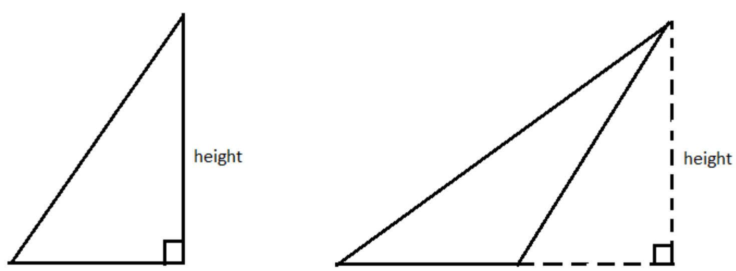

Figure 3: right triangle's height and obtuse triangle's height

In the next example, you need to solve an equation based on a triangle's area formula.

[**Example 2**] A triangle covers 20 square millimeters. Its base is 5 millimeters. Find its height.

[**Solution**] Let the triangle's height be *h* millimeters. Plug the given numbers into the triangle area formula, we have:

$$
\begin{aligned}
A &= \frac{1}{2}(base)(height) \\
20 &= \frac{1}{2} \cdot 5h \\
\mathbf{2} \cdot 20 &= \mathbf{2} \cdot \frac{1}{2} \cdot 5h \\
40 &= 5h \\
\frac{40}{5} &= \frac{5h}{5} \\
8 &= h
\end{aligned}
$$

Solution: The triangle's height is 8 millimeters. Note that in the third row, we multipled both sides of the equation by 2 to get rid of the fraction $\frac{1}{2}$.

Area of Cirlces

Next, let's learn the famous circle area formula: $A = \pi r^2$. The only thing we need to be careful about is that sometimes the diameter is given, and we need to find the radius first.

[**Example 3**] A circle's diameter is 8 yards. Find this circle's area in terms of π, and then round the area to the hundredth place.

[**Solution**] We first find the circle's radius, which is half of its diameter—4 yards in this problems. Next, we use a circle's area formula:

$$A = \pi r^2 = \pi \cdot 4^2 = 16\pi \text{ yd}^2$$

Next, we use a scientific calculator to change the result to decimal, and then round to the hundredth place:

$$A = 16\pi = 16 \cdot 3.1415026... \approx 50.27 \text{ yd}^2$$

Solution: The circle's area is 16π yd^2 (accurate value), or approximately 50.27 yd^2.

In the next example, the area is given, and you are asked to find the circle's radius. We need to review the concept of "square root" first.

$0^2 = 0$	$\sqrt{0} = 0$
$1^2 = 1$	$\sqrt{1} = 1$
$2^2 = 4$	$\sqrt{4} = 2$
$3^2 = 9$	$\sqrt{9} = 3$
$4^2 = 16$	$\sqrt{16} = 4$
...	...
$10^2 = 100$	$\sqrt{100} = 10$

Square root does the opposite of square. If we know $r^2 = 100$, we use square root to find *r*'s value:

$$\begin{aligned} r^2 &= 100 \\ r &= \sqrt{100} \\ r &= 10 \end{aligned}$$

[**Example 4**] A circle's area is 100 square meters. Find this radius diameter. Round your answer to hundredth place.

[**Solution**] To find a circle's diameter, we need to find its radius. Plug $A = 100$ into a circle's area formula, we have:

$$\begin{aligned} A &= \pi r^2 \\ 100 &= \pi r^2 \\ \frac{100}{\pi} &= \frac{\pi r^2}{\pi} \\ \frac{100}{\pi} &= r^2 \\ \sqrt{\frac{100}{\pi}} &= r \\ 5.64 &\approx r \end{aligned}$$

Since a circle's diameter is twice its radius, we have:

$$d = 2r = 2 \cdot 5.64 = 11.28 \text{ meters}$$

Solution: The circle's diameter is approximately 11.28 meters.