Add and Subtract Integers

In the last lesson, we learned: negative times negative is positive. This rule only works for multiplication and division. The rules are different for addition and subtraction! For example: $(-1)+(-2)=-3$. See? Negative plus negative is NOT positive.

Add Integers

I will show two methods to add integers. You choose the method which makes more sense for you. We will calculate $(-1)+(-2)$.

Method 1: The first method is the traditional number line method. Recall that the right side is the positive direction, and the left side is negative direction.

Step 1: Locate the first number, -1, on the number line.

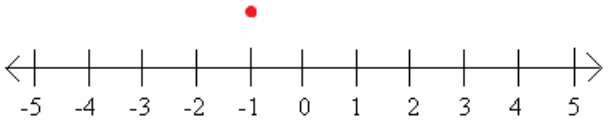


Figure 1: Locate -1 on a number line

Step 2: The second number is -2, meaning we will move to the left (negative direction) by two units.

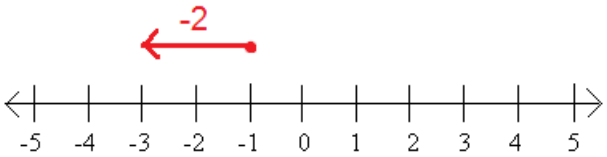


Figure 2: From -1, move to the left by 2 units.

Step 3: After the move, we reached the number -3 on the number line. This implies $(-1)+(-2)=-3$.

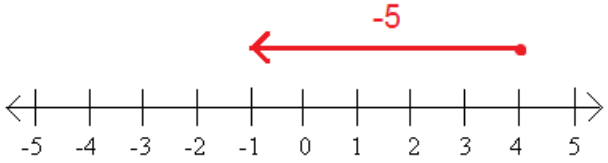
Method 2: The second method uses a money model. We deal with money every day, so most students can easily understand this method.

Say you are gambling. The first number is -1, meaning you lost \$1 in the first game. The second number is -2, meaning you lost \$2 in the second game. Altogether, you lost \$3, implying $(-1)+(-2)=-3$.

We will look at two more examples.

[**Example 1**] Calculate $4+(-5)$.

Method 1: On the number line, locate the number 4, and then move left by 5 units:



We reached -1 on the number line. This implies $4+(-5)=-1$.

Method 2:

- The first number is 4, implying you won $4 in the first game.
- The second number is -5, implying you lost $5 in the second game.
- Since you lost more money, the answer must be negative.
- Since you won some money and then lost some, it makes sense that you lost the difference of $4 and $5, which is $1.

This is why $4+(-5)=-1$.

[**Example 2**] Calculate $(-3)+3$.

Method 1: On the number line, locate the number -3, and then move right by 3 units:

We reached 0 on the number line. This implies $(-3)+3=0$.

Method 2: First we lost $3, and then we won $3 back. Altogether, we broke even, implying $(-3)+3=0$.

Subtract Positive Integers

Let's observe a pattern first:

$$3-2=1 \qquad 3+(-2)=1$$
$$4-3=1 \qquad 4+(-3)=1$$
$$4-1=3 \qquad 4+(-1)=3$$

We can see that the subtraction sign can be changed to "adding a negative." Remember this: the subtraction sign (minus) and the negative sign have the same functionalities! The earlier you understand this, the better.

Thus, when we calculate $4-9$, we will first change the problem from subtraction to addition: $4-9=4+(-9)$. Then we will use either the number line method or the money model to do addition.

[**Example 3**] Calculate $4-9$.

[**Solution**] $4-9=4+(-9)=-5$

Let's use the money model. We first won \$4, and then lost \$9. We lost more money, so the answer is negative. We lost the difference between \$4 and \$9, which is \$5. This is why the answer is -5.

[**Example 4**] Calculate $-4-9$.

[**Solution**] $-4-9=-4+(-9)=-13$

We first lost \$4, and then lost \$9. Altogether, we lost \$13. The answer is -13.

Subtract Negative Integers

The first step to calculate $3-(-2)$ is to change those two negative signs to plus: $3-(-2)=3+2$. Why?

Here is an easy way to help you make sense of this change: If I do NOT NOT like football, it actually means I like football. Double negation cancel one another. This is why $3-(-2)=3+2$.

If this is not "mathematical" enough for you, read on. Let's observe a pattern first:

$$-2=-1\cdot 2$$
$$-3=-1\cdot 3$$
$$-4=-1\cdot 4$$
$$\ldots$$

We can see the negative sign is equivalent to "negative one times".

Earlier, we also learned that the minus sign can be changed to "adding a negative." So we have:

$$3-(-2)=3+-(-2)=3+(-1)\cdot(-1)\cdot 2=3+1\cdot 2=3+2$$

Basically, those two minus signs mean "-1 times -1", which is positive 1. If this is confusing, you can simply understand $3-(-2)=3+2$ by double negation.

The next example puts together everything we learned so far.

[**Example 5**] Calculate $-3-15-(-29)-4$

[**Solution**] The first step is to seek "minus negative" and change it to plus:

$$-3-15-(-29)-4=-3-15+29-4$$

Next, change "minus" to "plus negative":

$$-3-15+29-4=-3+(-15)+29+(-4)$$

Next, do addition step by step. Do $-3+(-15)$ first. We lost \$3, and then lost \$15. Altogether, we lost \$18, so $-3+(-15)=-18$. We have:

$$-3+(-15)+29+(-4)=-18+29+(-4)$$

Next step is $-18+29$. We lost \$18, and then won \$29. We won more, so the result is positive. We won the difference between 18 and 29, which is 11. So $-18+29=11$. We have:

$$-18+29+(-4)=11+(-4)$$

Finally, we won \$11 and lost \$4. Altogether, we won \$7. We have:

$$11+(-4)=7$$

Solution: $-3-15-(-29)-4=7$