6.3 Percent Word Problems

We often encounter the following problems in real life:

- You make $15.00 an hour, and you just got a 5% raise! How much do you make now?
- Jose's dinner bill was $90.00, and he added $13.50 as a tip. How much percent of a tip did Jose give?
- Each vitamin supplement pill contains 54mg of Vitamin C, which is 60% of an adult's recommended daily Vitamin C intake. How many mg of Vitamin C should an adult consume every day?

After this lesson, you will be able to answer these questions.

In the last lesson, we learned three types of percent problems:

- **Type I:** What is 40% of 20?
- **Type II:** 8 is what percent of 20?
- **Type III:** 8 is 40% of what?

Each word problem can be boiled down to one of these three types.

6.3.1 Type I Percent Word Problems

Example 6.3.1 You make $15.00 an hour, and you just got a 5% raise! How much do you make now?

Solution We will use the Percent Formula to solve this problem.

Method 1: This problem boils down to this question: What is 5% of 15? This is Type I percent problem. Assume x is 5% of 15. We will write down the "Percent Formula" and the problem right next to each other:

$$
\begin{aligned}
3 &= 50\% \cdot 6 \\
x &= 5\% \cdot 15
\end{aligned}
$$

Next, we can solve for x in the equation. In this type of problems, x happens to be alone on one side of the equal sign, so all we need to do is to do the calculation on the other side of the equal sign. Remember that 5% = 0.05. We have:

$$
\begin{aligned}
x &= 5\% \cdot 15 \\
x &= 0.05 \cdot 15 \\
x &= 0.75
\end{aligned}
$$

Since 5% of $15.00 is $0.75, after the raise, you are paid $15.00+$0.75 =$15.75 per hour.

Method 2: After a 5% raise, your new pay is 105% of your old pay. Now this problem boils down to this question: What is 105% of 15? This is Type I percent problem. Assume x is 105% of 15. We can solve this equation:

$$
\begin{aligned}
x &= 105\% \cdot 15 \\
x &= 1.05 \cdot 15 \\
x &= 15.75
\end{aligned}
$$

∎

Conclusion: After the raise, you are paid $15.75 per hour.

6.3.2 Type II Percent Word Problem

Example 6.3.2 Jose's dinner bill was $90.00, and he added $13.50 as a tip. How much percent of a tip did Jose give?

Solution We need to find $13.50 is what percent of the $90.00. This is a Type II percent problem. We will use proportion to solve this problem.

Assume 13.5 is x% of 90. We can rephrase this sentence as: 13.5 out of 90 is like x out of 100. Here is the key: The number following the word "of" corresponds to 100. Now we can set up and solve a proportion:

$$\frac{13.5}{90} = \frac{x}{100}$$
$$90x = 13.5 \cdot 100$$
$$90x = 1350$$
$$\frac{90x}{90} = \frac{1350}{90}$$
$$x = 15$$

∎

Conclusion: Jose gave 15% tip.

6.3.3 Type III Percent Problem

Example 6.3.3 Each vitamin supplement pill contains 54mg of Vitamin C, which is 60% of an adult's recommended daily Vitamin C intake. How many mg of Vitamin C should an adult consume every day?

Solution If an adult takes one pill, he/she has taken 54mg of Vitamin C, or 60% of a day's recommended Vitamin C intake. This problem can be boiled down to this problem: 54 is 60% of what? This is a Type III percent problem.

We will use multiplication/division to solve this problem. No variable (x) is involved in this method. The key is to write down a simply example on scratch paper, and then put numbers in their corresponding places.

To find "3 is 50% of what", we do:
$$3 \div 0.5 = 6$$

Similarly, to find "54 is 60% of what", we do:
$$54 \div 0.6 = 90$$

∎

Conclusion: An adult should consume 90 mg of Vitamin C every day.

Example 6.3.4 Omar sells cars for a living. His monthly base pay is $1,200.00. On top of his base pay, Omar keeps 2.5% of his sales as commission. In a certain month, Omar took home a total of $2,950.00. How much in car sales did Omar make in that month?

Solution Omar took home a total of $2,950.00. Out of this amount, $1,200.00 was his base pay, so his commission pay was $2,950.00−$1,200.00 =$1,750.00.

The $1,750.00 was his commission, which was 2.5% of his sales in that month. Now this problem can be rephrased as: 1750 is 2.5% of what? This is a Type III percent problem. We will use the Percent Formula to solve this problem.

Assume 1750 is 2.5% of x. We will write down the "Percent Formula" and the problem right next to each other:

$$3 \quad = \quad 50\% \quad \cdot \quad 6$$
$$1750 \quad = \quad 2.5\% \quad \cdot \quad x$$

Next, we can solve for x in the equation:

$$1750 = 2.5\% \cdot x$$
$$1750 = 0.025x$$
$$\frac{1750}{0.025} = \frac{0.025x}{0.025}$$
$$70000 = x$$

■

Conclusion: Omar made $70,000.00 in car sales in that month.

6.3.4 Rounding

Sometimes we need to round numbers, like in the next example.

Example 6.3.5 A small town has 981 residents. Out of these residents, there are 45 Asians. What percent of the town's population are Asians? Round your answer to the hundredths place.

Solution This problem can be boiled down to this problem: 45 is what percent of 981? This is a Type II percent problem. We will use the Percent Formula to solve this problem. Assume 45 is x (as a percent) of 981.
We will write down the "Percent Formula" and the problem right next to each other:

$$
\begin{array}{ccccc}
3 & = & 50\% & \cdot & 6 \\
45 & = & x \text{ (as a percent)} & \cdot & 981
\end{array}
$$

Next, we can solve for x in the equation:

$$45 = x \cdot 981$$
$$45 = 981x$$
$$\frac{45}{981} = \frac{981x}{981}$$
$$0.0459 \approx x$$
$$4.59\% \approx x$$

■

Conclusion: Approximately 4.59% of the town's population are Asians.

6.3.5 Percent Problems involving Subtraction

In percent problems, keep in mind that the whole is always 100%. See the next example.

Example 6.3.6 In a county, 36.2% of registered voters are Democrats, 40.9% of registered voters are Republicans. The rest of registered voters are Independents. The county has a total of 12,000 registered voters. How many registered Independents live in this county?

Solution Out of the county's 100% registered voters, 36.2% are Democrats, and 40.9% are Republicans. This implies Independents make up $100\% - 36.2\% - 40.9\% = 22.9\%$ of all registered voters. Now the problem becomes: What is 22.9% of 12,000?
We will use multiplication/division to solve this problem. No variable (x) is involved in this method. The key is to write down a simple example on scratch paper, and then put numbers in their corresponding places.
To find "50% of 6", we do:
$$50\% \cdot 6 = 0.5 \cdot 6 = 3$$

Similarly, to find "22.9% of 12,000", we do:

$$22.9\% \cdot 12400 = 0.229 \cdot 12000 = 2748$$

∎

Conclusion: The county has 2,748 registered Independent voters.

6.3.6 **More than** 100%

Once you are done with a percent problem, quickly use mental math to check your solution and see whether it makes common sense.

Example 6.3.7 In a high school, the number of African American students is 120% of the number of Asian students. If there are 72 African American students, how many Asian students are there?

Solution It's given that the number of African American students is 120% of the number of Asian students. This implies there are more African American students than Asian students in this school. We will use this to quickly check our answer later.

We will use proportion to solve this problem. Assume the school has x Asian students. This problem boils down to this question: 72 is 120% of what? We can rephrase this sentence as: 72 out of x is like 120 out of 100. Here is the key: The number following the word "of" corresponds to 100. Now we can set up and solve a proportion:

$$\frac{72}{x} = \frac{120}{100}$$
$$120x = 72 \cdot 100$$
$$120x = 7200$$
$$\frac{120x}{120} = \frac{7200}{120}$$
$$x = 60$$

Conclusion: The school has 60 Asian students.

Our solution shows there are more African American students than Asian students in this school, which matches the given information we mentioned earlier. If the result shows more Asian students, we would know we made a mistake. ∎