

6.2 Percent Formula

In this lesson, we will learn 3 types of percent problems, and use 3 methods to solve each type of problems. You can choose your favorite method. It would be great if you can use all methods, so you can have a better understanding of percent problems.

We will handle the following 3 types of percent problems:

- **Type I:** What is 40% of 20?
- **Type II:** 8 is what percent of 20?
- **Type III:** 8 is 40% of what?

The Percent Formula is one popular method to solve the above problems.

6.2.1 Percent Formula

Many textbooks give the Percent Formula as:

$$\text{Part} = \text{Percent} \cdot \text{Whole}$$

or something similar. In this lesson, instead of memorizing a formula, I expect you to write out the following example on scratch paper.

Recall that the English word "of" can be translated into the multiplication symbol, in situations like "3 is $\frac{1}{2}$ of 6." You can verify this by doing:

$$\frac{1}{2} \cdot 6 = \frac{1}{2} \cdot \frac{6}{1} = \frac{6}{2} = 3$$

It's common sense that $\frac{1}{2} = 50\%$, so we have "3 is 50% of 6." Here is the percent formula:

$$3 = 50\% \cdot 6$$

Once we change 50% to 0.5, we can verify $0.5 \cdot 6 = 3$ with calculator.

When you need to use the Percent Formula, instead of writing down a formula, write down the example above. In later lessons, when I refer to the "Percent Formula", I mean:

$$3 = 50\% \cdot 6$$

There are three types of percent problems.

6.2.2 Type I Percent Problem

Example 6.2.1 What is 40% of 20?

Solution Method 1: Use Percent Formula Assume x is 40% of 20. We will write down the "Percent Formula" and the problem right next to each other:

$$\begin{aligned} 3 &= 50\% \cdot 6 \\ x &= 40\% \cdot 20 \end{aligned}$$

Next, we can solve for x in the equation. In this type of problems, x happens to be alone on one side of the equal sign, so all we need to do is to do the calculation on the other side of the equal sign. Remember that $40\% = 0.4$. We have:

$$\begin{aligned} x &= 40\% \cdot 20 \\ x &= 0.4 \cdot 20 \\ x &= 8 \end{aligned}$$

Conclusion: 8 is 40% of 20.

Method 2: Use Proportion Assume x is 40% of 20. We can rephrase this sentence as: x out of 20 is like 40 out of 100. Here is the key: The number following the word "of" corresponds to 100. Now we can set up and solve a proportion:

$$\begin{aligned}\frac{x}{20} &= \frac{40}{100} \\ 100x &= 20 \cdot 40 \\ 100x &= 800 \\ \frac{100x}{100} &= \frac{800}{100} \\ x &= 8\end{aligned}$$

Conclusion: 8 is 40% of 20.

Method 3: Use Multiplication/Division No variable (x) is involved in this method. The key is to write down a simple example on scratch paper, and then put numbers in their corresponding places.

To find "50% of 6", we do:

$$50\% \cdot 6 = 0.5 \cdot 6 = 3$$

Similarly, to find "40% of 20", we do:

$$40\% \cdot 20 = 0.4 \cdot 20 = 8$$

Conclusion: 8 is 40% of 20. ■

6.2.3 Type II Percent Problem

Example 6.2.2 8 is what percent of 20?

Solution Method 1: Use Percent Formula Assume 8 is x (as a percent) of 20. We will write down the "Percent Formula" and the problem right next to each other:

$$\begin{aligned}3 &= 50\% \cdot 6 \\ 8 &= x \text{ (as a percent)} \cdot 20\end{aligned}$$

Next, we can solve for x in the equation:

$$\begin{aligned}8 &= x \cdot 20 \\ 8 &= 20x \\ \frac{8}{20} &= \frac{20x}{20} \\ 0.4 &= x \\ 40\% &= x\end{aligned}$$

Conclusion: 8 is 40% of 20.

Method 2: Use Proportion Assume 8 is $x\%$ of 20. We can rephrase this sentence as: 8 out of 20 is like x out of 100. Here is the key: The number following the word "of"

corresponds to 100. Now we can set up and solve a proportion:

$$\begin{aligned}\frac{8}{20} &= \frac{x}{100} \\ 20x &= 8 \cdot 100 \\ 20x &= 800 \\ \frac{20x}{20} &= \frac{800}{20} \\ x &= 40\end{aligned}$$

Conclusion: 8 is 40% of 20.

Method 3: Use Multiplication/Division No variable (x) is involved in this method. The key is to write down a simple example on scratch paper, and then put numbers in their corresponding places.

To find "3 is what percent of 6", we do:

$$3 \div 6 = 0.5 = 50\%$$

Similarly, to find "8 is what percent of 20", we do:

$$8 \div 20 = 0.4 = 40\%$$

Conclusion: 8 is 40% of 20. ■

6.2.4 Type III Percent Problem

Example 6.2.3 8 is 40% of what?

Solution Method 1: Use Percent Formula Assume 8 is 40% of x . We will write down the "Percent Formula" and the problem right next to each other:

$$\begin{aligned}3 &= 50\% \cdot 6 \\ 8 &= 40\% \cdot x\end{aligned}$$

Next, we can solve for x in the equation:

$$\begin{aligned}8 &= 40\% \cdot x \\ 8 &= 0.4x \\ \frac{8}{0.4} &= \frac{0.4x}{0.4} \\ 20 &= x\end{aligned}$$

Conclusion: 8 is 40% of 20.

Method 2: Use Proportion Assume 8 is 40% of x . We can rephrase this sentence as: 8 out of x is like 40 out of 100. Here is the key: The number following the word "of" corresponds to 100. Now we can set up and solve a proportion:

$$\begin{aligned}\frac{8}{x} &= \frac{40}{100} \\ 40x &= 8 \cdot 100 \\ 40x &= 800 \\ \frac{40x}{40} &= \frac{800}{40} \\ x &= 20\end{aligned}$$

Conclusion: 8 is 40% of 20.

Method 3: Use Multiplication/Division No variable (x) is involved in this method. The key is to write down a simple example on scratch paper, and then put numbers in their corresponding places.

To find "3 is 50% of what", we do:

$$3 \div 0.5 = 6$$

Similarly, to find "8 is 40% of what", we do:

$$8 \div 0.4 = 20$$

Conclusion: 8 is 40% of 20. ■

6.2.5 Rounding

Sometimes we need to round numbers, like in the next example.

Example 6.2.4 45 is what percent of 981? Round your answer to two decimal places.

Solution This is a Type II percent problem. We will use the Percent Formula to solve this problem. Assume 45 is x (as a percent) of 981.

We will write down the "Percent Formula" and the problem right next to each other:

$$\begin{array}{rcl} 3 & = & 50\% \cdot 6 \\ 45 & = & x \text{ (as a percent)} \cdot 981 \end{array}$$

Next, we can solve for x in the equation:

$$\begin{array}{r} 45 = x \cdot 981 \\ 45 = 981x \\ \frac{45}{981} = \frac{981x}{981} \\ 0.0459 \approx x \\ 4.59\% \approx x \end{array}$$

Conclusion: 45 is approximately 4.59% of 981. ■

6.2.6 More than 100%

The rules are the same when we deal with percents bigger than 100%. Let's look at a few examples. To save space, we will solve these problems only with the Percent Formula.

Example 6.2.5 What is 140% of 20?

Solution We know the solution must be bigger than 20, because the percent is bigger than 100%. Assume x is 140% of 20. We will write down the "Percent Formula" and the problem right next to each other:

$$\begin{array}{rcl} 3 & = & 50\% \cdot 6 \\ x & = & 140\% \cdot 20 \end{array}$$

Next, we can solve for x in the equation. In this type of problems, x happens to be alone on one side of the equal sign, so all we need to do is to do the calculation on the other side of the equal sign. We have:

$$\begin{array}{r} x = 140\% \cdot 20 \\ x = 1.4 \cdot 20 \\ x = 28 \end{array}$$

Conclusion: 28 is 140% of 20.

Example 6.2.6 28 is what percent of 20?

Solution We know the solution must be bigger than 100%, because 28 is bigger than 20. Assume 28 is x (as a percent) of 20. We will write down the "Percent Formula" and the problem right next to each other:

$$\begin{array}{rcl} 3 & = & 50\% \cdot 6 \\ 28 & = & x \text{ (as a percent)} \cdot 20 \end{array}$$

Next, we can solve for x in the equation:

$$\begin{aligned} 28 &= x \cdot 20 \\ 28 &= 20x \\ \frac{28}{20} &= \frac{20x}{20} \\ 1.4 &= x \\ 140\% &= x \end{aligned}$$

Conclusion: 28 is 140% of 20. ■

Example 6.2.7 28 is 140% of what?

Solution We know the solution must be smaller than 28, because the percent is bigger than 100%. Assume 28 is 140% of x . We will write down the "Percent Formula" and the problem right next to each other:

$$\begin{array}{rcl} 3 & = & 50\% \cdot 6 \\ 28 & = & 140\% \cdot x \end{array}$$

Next, we can solve for x in the equation:

$$\begin{aligned} 28 &= 140\% \cdot x \\ 28 &= 1.4x \\ \frac{28}{1.4} &= \frac{1.4x}{1.4} \\ 20 &= x \end{aligned}$$

Conclusion: 28 is 140% of 20. ■