MODULE 5

It's critical to understand rate and proportion, as we use the concept every day. For example, if it takes $2\frac{1}{4}$ cups of flour to make three servings of food, how many cups of flour should be used to make eight servings? In super market, if 6 oz of coffee costs 7.99, while 9 oz of coffee costs 9.99, which choice is cheaper?

5.1 Rate and Ratio

5.1.1 Rate

In a rate, we always see the word "per", "each", "every" or simply the number 1. We use division to calculate rate. It's important to include units in the calculation. Let's look at a few examples.

Example 5.1.1 A car drove 150 miles in 6 hours. What's the car's speed in miles per hour?

Solution We use division to find the rate of change (speed in this case):

$$\frac{150 \text{ miles}}{6 \text{ hours}} = 25 \text{ miles/hour}$$

The car's speed is 25 miles per hour. ■

Example 5.1.2 A car drove 150 miles in 6 hours. How long does it take the car to travel each mile?

Solution We use division to find the rate of change:

$$\frac{6 \text{ hours}}{150 \text{ miles}} = 0.04 \text{ hour/mile}$$

It takes the car 0.04 hour to drive each mile. Later we will learn that 0.04 hour is 144 seconds. ■

We can see 25 miles/hour and 0.04 hour/mile are equivalent.

Compare those two examples above, you can see why it's important to include units in the calculation of rate. In real life, speed is regularly used, but sometimes we need to use the other rate: 0.04 hours per mile.

5.1.2 Problem Solving with Rate

When we use rate to solve problems, the key is to include units in the calculation. Let's look an example:

Example 5.1.3 A car can drive 25 miles per hour.

1. How long does it take the car to drive 300 miles?

2. How far can the car go in 30 hours?

Solution We could use multiplication and division to solve this problem. However, we will use fraction multiplication, which is an important skill later when we study unit conversions. The rate is given as 25 miles per hour. This can be written in two ways: $\frac{25 \text{ miles}}{1 \text{ hour}}$ or $\frac{1 \text{ hour}}{25 \text{ miles}}$.

1. How long does it take the car to drive 300 miles?
   We need to use the rate $\frac{1 \text{ hour}}{25 \text{ miles}}$, so the unit "miles" will cancel:

$$\begin{aligned}
&300 \text{ miles} \cdot \frac{1 \text{ hour}}{25 \text{ miles}} \\
&= \frac{300 \text{ miles}}{1} \cdot \frac{1 \text{ hour}}{25 \text{ miles}} \\
&= \frac{300}{1} \cdot \frac{1 \text{ hour}}{25} \\
&= \frac{300 \cdot 1}{1 \cdot 25} \text{ hours} \\
&= \frac{300}{25} \text{ hours} \\
&= 12 \text{ hours}
\end{aligned}$$

   It takes the car 12 hours to drive 300 miles.
2. How far can the car go in 30 hours?
   We need to use the rate $\frac{25 \text{ miles}}{1 \text{ hour}}$, so the unit "hours" will cancel:

$$\begin{aligned}
&30 \text{ hours} \cdot \frac{25 \text{ miles}}{1 \text{ hour}} \\
&= \frac{30 \text{ hours}}{1} \cdot \frac{25 \text{ miles}}{1 \text{ hour}} \\
&= \frac{30}{1} \cdot \frac{25 \text{ miles}}{1} \\
&= 30 \cdot 25 \text{ miles} \\
&= 750 \text{ miles}
\end{aligned}$$

   The car can drive 750 miles in 30 hours. ■

From the first two examples in this lesson, we learned that 25 miles/hour and 0.04 hour/mile are equivalent. In the next example, we will solve the same problem with the rate 0.04 hour/mile.

Example 5.1.4 It takes a car 0.04 hour to drive a mile.

1. How long does it take the car to drive 300 miles?
2. How far can the car go in 30 hours?

Solution The rate is given as 0.04 hour per mile. This can be written in two ways: $\frac{0.04 \text{ hour}}{1 \text{ mile}}$ or $\frac{1 \text{ mile}}{0.04 \text{ hour}}$.

1. How long does it take the car to drive 300 miles?
   We need to use the rate $\frac{0.04 \text{ hour}}{1 \text{ mile}}$, so the unit "mile" will cancel:

$$\begin{aligned}
&300 \text{ miles} \cdot \frac{0.04 \text{ hour}}{1 \text{ mile}} \\
&= \frac{300 \text{ miles}}{1} \cdot \frac{0.04 \text{ hour}}{1 \text{ mile}} \\
&= \frac{300}{1} \cdot \frac{0.04 \text{ hour}}{1} \\
&= 300 \cdot 0.04 \text{ hours} \\
&= 12 \text{ hours}
\end{aligned}$$

It takes the car 12 hours to drive 300 miles.

2. How far can the car go in 30 hours?
We need to use the rate $\frac{1 \text{ mile}}{0.04 \text{ hour}}$, so the unit "hour" will cancel:

$$\begin{aligned}&30 \text{ hours} \cdot \frac{1 \text{ mile}}{0.04 \text{ hour}}\\ =&\frac{30 \text{ hours}}{1} \cdot \frac{1 \text{ mile}}{0.04 \text{ hour}}\\ =&\frac{30}{1} \cdot \frac{1 \text{ miles}}{0.04}\\ =&\frac{30 \cdot 1}{1 \cdot 0.04} \text{ miles}\\ =&\frac{30}{0.04} \text{ miles}\\ =&750 \text{ hours}\end{aligned}$$

The car can drive 750 miles in 30 hours. ■

5.1.3 Ratio

Ratio is very similar to rate, except it doesn't have unit. Let's look at a few examples.

- If a class has 30 male students and 10 female students, the ratio of male to female students is $\frac{30 \text{ people}}{10 \text{ people}} = \frac{30}{10} = \frac{3}{1}$. We can also say the ratio of male to female students is 3 : 1, or "3 to 1".
- If a class has 30 male students and 10 female students, the ratio of female to male students is $\frac{10 \text{ people}}{30 \text{ people}} = \frac{10}{30} = \frac{1}{3}$. We can also say the ratio of female to male students is 1 : 3, or "1 to 3".
- If Tom makes \$15.00 per hour and Jerry makes \$12.00 per hour, the ratio of Tom and Jerry's income is $\frac{15 \text{ dollars/hour}}{12 \text{ dollars/hour}} = \frac{15}{12} = \frac{5}{4}$. We can also say the ratio of Tom and Jerry's income is 5 : 4, or "5 to 4".
- If Tom makes \$15.00 per hour and Jerry makes \$12.00 per hour, the ratio of Jerry and Tom's income is $\frac{12 \text{ dollars/hour}}{15 \text{ dollars/hour}} = \frac{12}{15} = \frac{4}{5}$. We can also say the ratio of Jerry and Tom's income is 4 : 5, or "4 to 5".

Note that the units always cancel in a ratio. If the units don't cancel, like in 25 miles/hour, then it's called a rate, not a ratio. That's the major difference between rate and ratio.

5.1.4 Problem Solving with Ratio

When we use ratio to solve word problems, it's important not only to include units, but also the concepts. Let's look at some examples.

Example 5.1.5 A restaurant's expense of labor cost to food cost is in the ratio of 8 : 3.

1. In one month, if the restaurant spent \$2,000.00 in labor cost, how much did it spend on food cost?
2. In another month, if the restaurant spent \$600.00 in food cost, how much did it spend on labor cost?

Solution The ratio of labor cost to food cost is given as 8 : 3. We can write it as either $\frac{8 \text{ dollars in labor cost}}{3 \text{ dollars in food cost}}$ or $\frac{3 \text{ dollars in food cost}}{8 \text{ dollars in labor cost}}$.

1. In one month, if the restaurant spent \$2,000.00 in labor cost, how much did it spend on food cost?

We need to use the ratio $\frac{3 \text{ dollars in food cost}}{8 \text{ dollars in labor cost}}$, so the unit "dollars in labor cost" will cancel:

$$\begin{aligned}
& 2000 \text{ dollars in labor cost} \cdot \frac{3 \text{ dollars in food cost}}{8 \text{ dollars in labor cost}} \\
&= \frac{2000 \text{ dollars in labor cost}}{1} \cdot \frac{3 \text{ dollars in food cost}}{8 \text{ dollars in labor cost}} \\
&= \frac{2000}{1} \cdot \frac{3 \text{ dollars in food cost}}{8} \\
&= \frac{2000 \cdot 3}{1 \cdot 8} \text{ dollars in food cost} \\
&= \frac{6000}{8} \text{ dollars in food cost} \\
&= 750 \text{ dollars in food cost}
\end{aligned}$$

In one month, if the restaurant spent \$2,000.00 in labor cost, it spent \$750 on food cost.

2. In another month, if the restaurant spent \$600.00 in food cost, how much did it spend on labor cost?
We need to use the ratio $\frac{8 \text{ dollars in labor cost}}{3 \text{ dollars in food cost}}$, so the unit "dollars in food cost" will cancel:

$$\begin{aligned}
& 600 \text{ dollars in food cost} \cdot \frac{8 \text{ dollars in labor cost}}{3 \text{ dollars in food cost}} \\
&= \frac{600 \text{ dollars in food cost}}{1} \cdot \frac{8 \text{ dollars in labor cost}}{3 \text{ dollars in food cost}} \\
&= \frac{600}{1} \cdot \frac{8 \text{ dollars in labor cost}}{3} \\
&= \frac{600 \cdot 8}{1 \cdot 3} \text{ dollars in labor cost} \\
&= \frac{4800}{3} \text{ dollars in labor cost} \\
&= 1600 \text{ dollars in labor cost}
\end{aligned}$$

In another month, if the restaurant spent \$600.00 in food cost, it spent \$1,600 on labor cost. ■

We can see why we need to write more than units in equations involving ratios—otherwise all units would be "dollars" in the above example, which would be confusing.