4.5 Square Root

4.5.1 Definition of Square Root

Let's review the definition of square:

$$\begin{aligned}
0^2 &= 0 \cdot 0 = 0 \\
1^2 &= 1 \cdot 1 = 1 \\
2^2 &= 2 \cdot 2 = 4 \\
3^2 &= 3 \cdot 3 = 9 \\
9^2 &= 9 \cdot 9 = 81
\end{aligned}$$

Again, it's critical to memorize the following square numbers:

$$0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144$$

Square root is the inverse operation of square. For example, if we want to know which number squared gives the number 4, we write $\sqrt{4}$. Since $2^2 = 4$, we have $\sqrt{4} = 2$. Here are a few more examples:

$$\begin{aligned}
\sqrt{0} &= 0 \quad \text{as } 0^2 = 0 \\
\sqrt{1} &= 1 \quad \text{as } 1^2 = 1 \\
\sqrt{4} &= 2 \quad \text{as } 2^2 = 4 \\
\sqrt{9} &= 3 \quad \text{as } 3^2 = 9 \\
\sqrt{81} &= 9 \quad \text{as } 9^2 = 81
\end{aligned}$$

Most of the time, the square root of an integer is an irrational decimal. We use calculators to find the square root of such numbers:

$$\begin{aligned}
\sqrt{2} &= 1.414... \\
\sqrt{3} &= 1.732... \\
\sqrt{1000} &= 31.622...
\end{aligned}$$

4.5.2 Square Root of Fractions Involving Perfect Squares

Let's look at a few examples:

$$\begin{aligned}
\sqrt{\frac{1}{4}} &= \frac{1}{2} \quad \text{as } \left(\frac{1}{2}\right)^2 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \\
\sqrt{\frac{1}{9}} &= \frac{1}{3} \quad \text{as } \left(\frac{1}{3}\right)^2 = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9} \\
\sqrt{\frac{4}{9}} &= \frac{2}{3} \quad \text{as } \left(\frac{2}{3}\right)^2 = \frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9}
\end{aligned}$$

To calculate the square root of a fraction, like $\sqrt{\frac{4}{9}}$, we need to take the square root of both the numerator and denominator:

$$\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$$

4.5.3 Square Root of Decimals Involving Square Numbers

Let's review something we learned earlier:

- When we calculate $0.2 \cdot 0.2$, first we do $2 \cdot 2 = 4$. From $2 \cdot 2$ to $0.2 \cdot 0.2$, the decimal point moved to the left twice in total, so we move the decimal point of 4 to the left twice and have $0.2 \cdot 0.2 = 0.04$.
- When we calculate $0.11 \cdot 0.11$, first we do $11 \cdot 11 = 121$. From $11 \cdot 11$ to $0.11 \cdot 0.11$, the decimal point moved to the left four times in total, so we move the decimal point of 121 to the left four times and have $0.11 \cdot 0.11 = 0.0121$.

Now let's look at a few examples of square root involving decimals:

$$\begin{aligned} \sqrt{0.04} &= 0.2 \quad \text{as } 0.2^2 = 0.2 \cdot 0.2 = 0.04 \\ \sqrt{1.14} &= 1.2 \quad \text{as } 1.2^2 = 1.2 \cdot 1.2 = 1.14 \\ \sqrt{0.0004} &= 0.02 \quad \text{as } 0.02^2 = 0.02 \cdot 0.02 = 0.0004 \end{aligned}$$

We could summarize a rule here. However, it's better to jot down a few numbers on scratch paper when calculating square root of decimals. For example, to calculate $\sqrt{0.0081}$, recognize that $9^2 = 81$, so we know the the answer could be 0.9, 0.09 or 0.009. Since $0.09^2 = 0.09 \cdot 0.09 = 0.0081$, we know $\sqrt{0.0081} = 0.09$

4.5.4 Square Root of Other Decimals

Most of the time, the square root of a decimal is an irrational decimal. We use calculators to find the square root of such numbers:

$$\begin{aligned} \sqrt{12.1} &= 3.4785... \\ \sqrt{0.1} &= 0.3162... \\ \sqrt{0.4} &= 0.6324... \end{aligned}$$

Compare the square root of these two numbers:

$$\begin{aligned} \sqrt{0.04} &= 0.2 \\ \sqrt{0.4} &= 0.6324... \end{aligned}$$

4.5.5 Square Root of Negative Numbers

When we evaluate $\sqrt{9}$, we are looking for a number whose square is 9. Since $3^2 = 9$, we have $\sqrt{9} = 3$.

How about $\sqrt{-9}$? We are looking for a number whose square is -9. Well, let's try -3: We have $(-3)^2 = (-3)(-3) = 9$, so -3 is not the square root of -9.

We cannot find such a number, because the square of any negative number is positive! Since we cannot find the square root of -9, we say $\sqrt{-9}$ doesn't exist, or *undefined*.