4.3 Multiply/Divide Decimals

In this lesson, we will learn how to multiply/divide decimals. Again, please refrain from using calculator in this section (actually in this whole course).

4.3.1 Multiply/Divide by Power of 10

Recall that for the number 12, we could write it as 12.0, 12.00 or 12.000, as the value doesn't change.
Let's observe a pattern. In the following numbers, I added trailing zeroes and decimal point to the end of integers in order to find the pattern:

$$12.0 \cdot 10 = 120.$$
$$12.00 \cdot 100 = 1200.$$
$$12.000 \cdot 1000 = 12000.$$

Here is the pattern: **When we multiply a number by a power of** 10**, like** $10, 100, 1000$**..., we move the decimal point to the right as many times as the number of zeroes.**
A similar pattern exists for division:

$$12000. \div 10 = 1200.$$
$$12000. \div 100 = 120.$$
$$12000. \div 1000 = 12.$$

Here is the pattern: **When we divide a number by a power of** 10**, like** $10, 100, 1000$**..., we move the decimal point to the left as many times as the number of zeroes.**
Don't try to memorize the patterns without understanding. If you forget in which direction the decimal point will move, on scratch paper, do $12. \cdot 10 = 120.$ and $120. \div 10 = 12.$, and then you can quickly recall the patterns.
The same patterns work for decimals. Here are a few examples:

$123.456 \cdot 10 = 1234.56$	Decimal point moves to the right once.
$123.456 \cdot 100 = 12345.6$	Decimal point moves to the right twice.
$123.456 \div 10 = 12.3456$	Decimal point moves to the left once.
$123.456 \div 100 = 1.23456$	Decimal point moves to the left twice.

Sometimes we need to fill in zeroes:

$1.2 \cdot 100 = 120$	Decimal point moves to the right twice.
$1.2 \cdot 1000 = 1200$	Decimal point moves to the right three times.
$1.2 \div 10 = 0.12$	Decimal point moves to the left once.
$1.2 \div 100 = 0.012$	Decimal point moves to the left twice.
$1.2 \div 1000 = 0.0012$	Decimal point moves to the left three times.

4.3.2 Moving Decimal Point in Multiplication

It's given that $12 \cdot 12 = 144$. What's the product of $12 \cdot 1.2$?
From the pattern we learned earlier, we know $1.2 = 12 \div 10$. So we have:

$$\begin{aligned} & 12 \cdot 1.2 \\ &= 12 \cdot 12 \div 10 \\ &= 144 \div 10 \\ &= 14.4 \end{aligned}$$

Without giving details, with the same method, we can calculate:

$$12 \cdot 0.12 = 1.44$$
$$12 \cdot 0.012 = 0.144$$
$$1.2 \cdot 1.2 = 1.44$$
$$0.12 \cdot 12 = 1.44$$
$$0.12 \cdot 1.2 = 0.144$$

Instead of writing a long sentence to summarize the pattern, I will simply use an example. It's given $12 \cdot 12 = 144$. When we calculate $0.12 \cdot 1.2$, observe that the decimal point moved to the left for a total of three times, compared with $12 \cdot 12$. So, we need to move the decimal point of 144 to the left three times, and we have $0.12 \cdot 1.2 = 0.144$.
The pattern can be applied when we move the decimal point to the left or right. Let's look at a few examples.

Example 4.3.1 Given $123 \cdot 45 = 5535$, calculate $1.23 \cdot 45000$.

Solution

- From 123 to 1.23, the decimal point moved to the left twice.
- From 45 to 45000, the decimal point moved to the right three times.
- Altogether, the decimal point moved to the right once.
- We move 5535's decimal point to the right once, and we have: $1.23 \cdot 45000 = 55350$. ■

Example 4.3.2 Given $123 \cdot 45 = 5535$, calculate $1.23 \cdot 4500$.

Solution

- From 123 to 1.23, the decimal point moved to the left twice.
- From 45 to 4500, the decimal point moved to the right twice.
- Altogether, the decimal point didn't move.
- The decimal point of the product 5535 didn't move, and we have: $1.23 \cdot 4500 = 5535$. ■

4.3.3 Moving Decimal Point in Division

There are similar rules of moving the decimal point when we do divisions. Let me explain the rules with fractions.
Recall that for a fraction, if we multiply or divide the same number in both the numerator and denominator, the fraction's value doesn't change. For example:

$$\frac{1}{2} = \frac{1 \cdot 2}{2 \cdot 2} = \frac{2}{4}$$
$$\frac{2}{4} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2}$$

Similarly, we have:

$$\frac{120}{20} = \frac{120 \cdot 10}{20 \cdot 10} = \frac{1200}{200}$$
$$\frac{120}{20} = \frac{120 \div 10}{20 \div 10} = \frac{12}{2}$$

Since the fraction line is the same as the division symbol, we can rewrite our observation as:

$$120 \div 20 = 1200 \div 200$$
$$120 \div 20 = 12 \div 2$$

The rule is: **In a division, if we move the decimal point in both numbers to the same direction for the same number of times, the quotient doesn't change.** For example:

$1.23 \div 0.3 = 12.3 \div 3$	Decimal point in both numbers moved to the right once.
$1.23 \div 0.3 = 123 \div 30$	Decimal point in both numbers moved to the right twice.
$1.23 \div 0.3 = 0.123 \div 0.03$	Decimal point in both numbers moved to the left once.

We will learn how to use this rule in the following examples.

Example 4.3.3 Calculate $20 \div 0.4$

Solution

- We need to change 0.4 to an integer by moving the decimal point to the right once.
- By the rule we learned above, we need to move the decimal point of both 20 and 0.4 to the right once, and we have $20 \div 0.4 = 200 \div 4$
- Now the division is easy. We have: $20 \div 0.4 = 200 \div 4 = 50$ ■

Example 4.3.4 Calculate $0.12 \div 0.004$

Solution

- We need to change 0.004 to an integer by moving the decimal point to the right three times.
- By the rule we learned above, we need to move the decimal point of both 0.12 and 0.004 to the right three times, and we have $0.12 \div 0.004 = 120 \div 4$
- Now the division is easy. We have: $0.12 \div 0.004 = 120 \div 4 = 30$ ■