MODULE 4

4.1 Introduction to Decimals

We use decimals every day as we buy things. For example, each pound of apple costs \$1.99. In this lesson, we will learn the definition of decimals.

4.1.1 Definition of Decimals

The best way to understand decimals is to think of money, because we deal with money every day. For example, 1.99 means 1 dollar and 99 cents.

4.1.2 Types of Decimals

There are a few types of decimals:

- **terminating decimal**: A terminating decimal has a limited number of places, like $0.1, 1.23, 1.2345$.
- **repeating decimal**: A repeating decimal goes on forever repeating the same digits again and again, like $0.111..., 4.121212..., 5.3124124124....$ A repeating decimal can be re-written with a bar over the repeating digits:

$$
\begin{aligned}
0.111... &= 0.\overline{1} \\
4.121212... &= 0.\overline{12} \\
5.3124124124... &= 5.3\overline{124}
\end{aligned}
$$

- **irrational decimal**: An irrational decimal goes on forever without repeating any pattern. Here are a few examples:

$$
\begin{aligned}
\pi &= 3.1415926... \\
\sqrt{2} &= 1.414... \\
\sqrt{3} &= 1.732...
\end{aligned}
$$

The decimal 0.03

Think of the number 0.03, which is 3 cents. Since each dollar has 100 cents, 0.03 represents 3 out of 100, or, in fraction, $\frac{3}{100}$. This fraction is read as "three hundredths".
This is why we call the second digit after the decimal point the *hundredths place.* We could read 0.03 either as "three hundredths," or simply "zero point zero three."
Here is one way to represent 0.03 graphically (the grid has 100 cells):

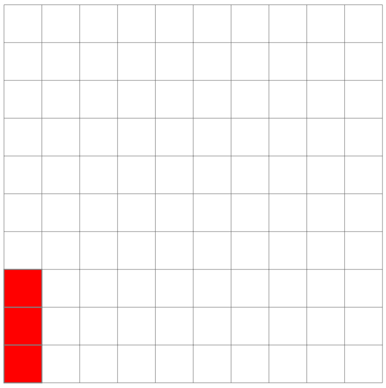

FIGURE 4.1: Graphic representation of 0.03

The decimal 0.3
Let's look at another number: 0.3. This number does not represent 3 cents, because earlier we learned 0.03 represents 3 cents. The difference is that, in 0.3, the number 3 is in the tenths place. In terms of money, 0.3 represents 30 cents. On a price mark, we usually write 0.3 as $0.30. This revealed an important property of decimals:
If we add the digit 0 to the end of a decimal (behind the decimal point), the decimal's value doesn't change. For example: $0.30 = 0.3$, $0.300 = 0.3$, $1.20 = 1.2$.
Note that $1.3 \neq 1.03$: the number 1.3 represents a dollar and 30 cents, while the number 1.03 represents a dollar and 3 cents.
The number 0.3 represents 3 out of 10, or, in fraction, $\frac{3}{10}$. This fraction is read as "three tenths".
This is why we call the digit after the decimal point the *tenths place.* We could read 0.3 either as "three tenths," or simply "zero point three."
Here is one way to represent 0.3 graphically:

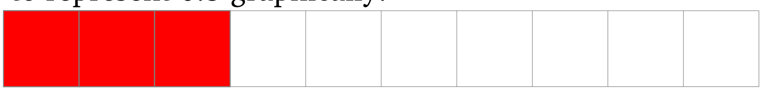

FIGURE 4.2: Graphic representation of 0.3

The decimal 0.003
Finally, let's look at 0.003. Earlier, we learned that 0.03 represents 3 cents. The number 0.003 is three tenth of a cent. If you look carefully at gas price next time you fill up, each gallon actually costs something like $3.159. Notice the last digit is always 9. Nine tenths of a cent is not a lot of money, but the profit adds up.
The number 0.003 represents 3 out of 1000, or, in fraction, $\frac{3}{1000}$. This fraction is read as "three thousandths".
This is why we call the third digit after the decimal point the *thousandths place.* We could read 0.003 either as "three thousandths," or simply "zero point zero zero three."
We will not represent 0.003 graphically here, because it's hard to draw a grid with 1,000 cells.

4.1.3 Read Decimals

For the decimal 1,234.5678, here are the names of each digit:

- 1 is in the thousands place, representing one thousand.
- 2 is in the hundreds place, representing two hundred.
- 3 is in the tens place, representing thirty.
- 4 is in the ones place, representing four.

- 5 is in the tenths place, representing five tenths (50 cents).
- 6 is in the hundredths place, representing six hundredths (6 cents).
- 7 is in the thousandths place, representing seven thousandths.
- 8 is in the ten-thousandths place, representing eight ten-thousandths.

Here are a few examples of how to read decimals:

- 12.3 reads: twelve and three tenths
- 12.34 reads: twelve and thirty four hundredths
- 12.345 reads: twelve and three hundred forty-five thousandths
- 12.3456 reads: twelve and three thousand four hundred fifty-six ten-thousandths
- 12.03 reads: twelve and three hundredths
- 12.003 reads: twelve and three thousandths

4.1.4 Decimals on Number Line

Here are a few examples on how to locate decimals on the number line:

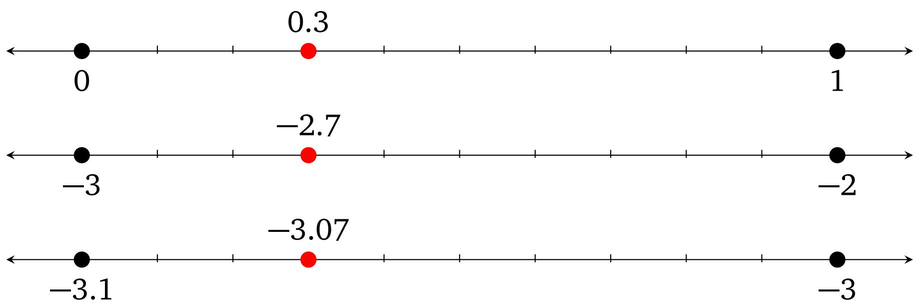

FIGURE 4.3: Decimals on the number line

4.1.5 Compare Decimals

Which one is bigger, 3.09 or 3.81? It's easy to compare if we think about money: three dollars and eighty-one cents is bigger than three dollars and nine cents. So we have:

$$3.09 < 3.81$$

So, when we compare decimals, we compare the tenths' place first, and then the hundredths' place. Even though the digit 9 in 3.09 is bigger than 8 in 3.81, the number 3.81 is bigger than 3.09.

We have the following comparisons:

$$1.29 < 1.30$$
$$4.29 > 1.30$$
$$3.04 < 3.4$$
$$0.031 > 0.009$$

Comparing negative numbers is "opposite:"

$$-1.29 > -1.30$$
$$-4.29 < -1.30$$
$$-3.04 > -3.4$$
$$-0.031 < -0.009$$

Finally, be careful when there are trailing zeroes:

$$1.10 = 1.1$$
$$0.03 = 0.030$$
$$-0.300 = -0.3$$

4.1.6 Round Decimals

Earlier, we learned how to round whole numbers. The concept of rounding decimals is the same. For example, to round 1.19 to an integer, we have $1.19 \approx 1$ because 1.19 is closer to 1 than 2. Let's look at a few examples:

Example 4.1.1 Round 1.295 to the tenths place.

Solution In 1.295, the tenths place is 2. The digit behind it is 9. Since 9 is bigger than 4, we round up:

$$1.295 \approx 1.3$$

■

Example 4.1.2 Round 1.245 to the tenths place.

Solution In 1.245, the tenths place is 2. The digit behind it is 4. Since 4 is smaller than 5, we don't round up:

$$1.245 \approx 1.2$$

■

Example 4.1.3 Round 1.295 to the hundredths place.

Solution In 1.295, the hundredths place is 9. The digit behind it is 5. Since 5 is bigger than 4, we round up:

$$1.295 \approx 1.30$$

■