3.8 Order of Operations involving Fractions

In this lesson, we will do a few order-of-operations problems involving fractions. We have learned the order of operations in earlier lessons:

P	(Parentheses)
E	(Exponent)
MD	(Multiplication and Division)
AS	(Addition and Subtraction)

FIGURE 3.15: Order of Operations

In this lesson, the rules didn't change, except fractions are involved. We will simply look at a few examples.

Example 3.8.1

$$\begin{aligned}
&\frac{5}{4}+(\frac{3}{4})2\\
=&\frac{5}{4}+\frac{3}{4}\cdot\frac{2}{1}\\
=&\frac{5}{4}+\frac{3}{4\div 2}\cdot\frac{2\div 2}{1}\\
=&\frac{5}{4}+\frac{3}{2}\cdot\frac{1}{1}\\
=&\frac{5}{4}+\frac{3}{2}\\
=&\frac{5}{4}+\frac{3\cdot 2}{2\cdot 2}\\
=&\frac{5}{4}+\frac{6}{4}\\
=&\frac{11}{4}
\end{aligned}$$

Without specified instruction, there is no need to change $\frac{11}{4}$ into $3\frac{3}{4}$.

Example 3.8.2

$$\begin{aligned}
&\frac{4}{3}-5(\frac{1}{9}-\frac{1}{6})\\
=&\frac{4}{3}-5(\frac{1\cdot 2}{9\cdot 2}-\frac{1\cdot 3}{6\cdot 3})\\
=&\frac{4}{3}-5(\frac{2}{18}-\frac{3}{18})\\
=&\frac{4}{3}-5(-\frac{1}{18})\\
=&\frac{4}{3}+5(\frac{1}{18}) && \text{(negative)}\cdot\text{(negative)} = \text{positive}\\
=&\frac{4}{3}+\frac{5}{1}\cdot\frac{1}{18}\\
=&\frac{4}{3}+\frac{5}{18}\\
=&\frac{4\cdot 6}{3\cdot 6}+\frac{5}{18}\\
=&\frac{24}{18}+\frac{5}{18}\\
=&\frac{29}{18}
\end{aligned}$$

In the step

$$\frac{4}{3}-5(-\frac{1}{18})$$

we treat the subtraction symbol as a negative symbol, so we have

$$\frac{4}{3}+(-5)\cdot(-\frac{1}{18})$$

Since (negative) $\cdot$ (negative) = positive, we have

$$\frac{4}{3}+(-5)\cdot(-\frac{1}{18})=\frac{4}{3}+5(\frac{1}{18})$$

Example 3.8.3 Compare the following example with the last one, and see the difference between parentheses and absolute value.

$$
\begin{aligned}
&\frac{4}{3}-5\left|\frac{1}{9}-\frac{1}{6}\right| \\
=&\frac{4}{3}-5\left|\frac{1\cdot 2}{9\cdot 2}-\frac{1\cdot 3}{6\cdot 3}\right| \\
=&\frac{4}{3}-5\left|\frac{2}{18}-\frac{3}{18}\right| \\
=&\frac{4}{3}-5\left|-\frac{1}{18}\right| \\
=&\frac{4}{3}-5(\frac{1}{18}) \\
=&\frac{4}{3}-\frac{5}{1}\cdot\frac{1}{18} \\
=&\frac{4}{3}-\frac{5}{18} \\
=&\frac{4\cdot 6}{3\cdot 6}-\frac{5}{18} \\
=&\frac{24}{18}-\frac{5}{18} \\
=&\frac{19}{18}
\end{aligned}
$$

Example 3.8.4 Compare these two examples:

$$
\begin{aligned}
&1-(\frac{2}{3})^2 \\
=&1-(\frac{2}{3})(\frac{2}{3}) \\
=&1-\frac{2\cdot 2}{3\cdot 3} \\
=&1-\frac{4}{9} \\
=&\frac{9}{9}-\frac{4}{9} \\
=&\frac{5}{9}
\end{aligned}
\qquad\qquad
\begin{aligned}
&1-(-\frac{2}{3})^2 \\
=&1-(-\frac{2}{3})(-\frac{2}{3}) \\
=&1-(\frac{2}{3})(\frac{2}{3}) \\
=&1-\frac{2\cdot 2}{3\cdot 3} \\
=&1-\frac{4}{9} \\
=&\frac{9}{9}-\frac{4}{9} \\
=&\frac{5}{9}
\end{aligned}
$$

Note that $(\frac{2}{3})^2=\frac{4}{9}$, and $(-\frac{2}{3})^2=\frac{4}{9}$.

Example 3.8.5 Compare these two examples:

$$
\begin{aligned}
& 1-(\frac{2}{3})^3 \\
&= 1-(\frac{2}{3})(\frac{2}{3})(\frac{2}{3}) \\
&= 1-\frac{2\cdot 2\cdot 2}{3\cdot 3\cdot 3} \\
&= 1-\frac{8}{27} \\
&= \frac{27}{27}-\frac{8}{27} \\
&= \frac{19}{27}
\end{aligned}
\qquad
\begin{aligned}
& 1-(-\frac{2}{3})^3 \\
&= 1-(-\frac{2}{3})(-\frac{2}{3})(-\frac{2}{3}) \\
&= 1+(\frac{2}{3})(\frac{2}{3})(\frac{2}{3}) \\
&= 1+\frac{2\cdot 2\cdot 2}{3\cdot 3\cdot 3} \\
&= 1+\frac{8}{27} \\
&= \frac{27}{27}+\frac{8}{27} \\
&= \frac{35}{27}
\end{aligned}
$$

Note that $1-(-\frac{2}{3})(-\frac{2}{3})(-\frac{2}{3})$ becomes $1+(\frac{2}{3})(\frac{2}{3})(\frac{2}{3})$ because each pair of negative symbols canceled each other.
Compare $(\frac{2}{3})^3=\frac{8}{27}$ and $(-\frac{2}{3})^3=-\frac{8}{27}$.
Earlier, we learned: $(\frac{2}{3})^2=\frac{4}{9}$, and $(-\frac{2}{3})^2=\frac{4}{9}$.
Instead of trying to memorize these results, understand that each pair of negative symbols cancel each other (in multiplication).