

3.7 Multiply/Divide Mixed Numbers

In this lesson, we will learn how to multiply and divide mixed numbers. The rule is very simple: Before we multiply/divide mixed numbers, we need to change them (like $1\frac{1}{2}$) to improper fractions (like $\frac{3}{2}$), and then use fraction multiplication/division rules we learned earlier.

3.7.1 Multiply/Divide Mixed Numbers

You might need to go back to review how to change a mixed number to an improper fraction.

Example 3.7.1

$$\begin{aligned} & 1\frac{1}{2} \cdot 3\frac{5}{6} \\ &= \frac{3}{2} \cdot \frac{23}{6} \\ &= \frac{3 \div 3}{2} \cdot \frac{23}{6 \div 3} \\ &= \frac{1}{2} \cdot \frac{23}{2} \\ &= \frac{1 \cdot 23}{2 \cdot 2} \\ &= \frac{23}{4} \\ &= 5\frac{3}{4} \end{aligned}$$

The last step is optional, depending on what the question asks. If the question doesn't ask for a mixed number answer, you could leave the answer as $\frac{23}{4}$.

Example 3.7.2

$$\begin{aligned} & 8\frac{1}{4} \div 3 \\ &= \frac{33}{4} \div \frac{3}{1} \\ &= \frac{33}{4} \cdot \frac{1}{3} \\ &= \frac{33 \div 3}{4} \cdot \frac{1}{3 \div 3} \\ &= \frac{11}{4} \cdot \frac{1}{1} \\ &= \frac{11}{4} \\ &= 2\frac{3}{4} \end{aligned}$$

3.7.2 Word Problems

Example 3.7.3 Each truck can hold $3\frac{1}{4}$ tons of sand. In a job, it took 24 full truck loads to transfer a certain amount of sand. How many tons of sand were transferred?

Solution This is obviously a multiplication problem:

$$\begin{aligned}
 & 3\frac{1}{4} \cdot 24 \\
 &= \frac{13}{4} \cdot \frac{24}{1} \\
 &= \frac{13}{4 \div 4} \cdot \frac{24 \div 4}{1} \\
 &= \frac{13}{1} \cdot \frac{6}{1} \\
 &= \frac{13 \cdot 6}{1 \cdot 1} \\
 &= \frac{78}{1} \\
 &= 78
 \end{aligned}$$

Conclusion: Those trucks transferred 78 tons of sand. ■

Example 3.7.4 In a running contest (with breaks), each runner will run 28 miles. A runner runs $5\frac{1}{4}$ miles per hour. How many hours will it take the runner to complete the race?

Solution In this problem, we need to find out how many $5\frac{1}{4}$ miles are in 28 miles, a division problem:

$$\begin{aligned}
 & 28 \div 5\frac{1}{4} \\
 &= \frac{28}{1} \div \frac{21}{4} \\
 &= \frac{28}{1} \cdot \frac{4}{21} \\
 &= \frac{28 \div 7}{1} \cdot \frac{4}{21 \div 7} \\
 &= \frac{4}{1} \cdot \frac{4}{3} \\
 &= \frac{16}{3} \\
 &= 5\frac{1}{3}
 \end{aligned}$$

Conclusion: It would take the runner $5\frac{1}{3}$ hours to complete the race.

Since each hour has 60 minutes, $\frac{1}{3}$ of an hour has 20 minutes, so it would take the runner 5 hours and 20 minutes to complete the race.

In this problem, it would not be good to say it would take the runner $\frac{16}{3}$ hours to complete the race. ■

Example 3.7.5 Find the area of the following rectangle.

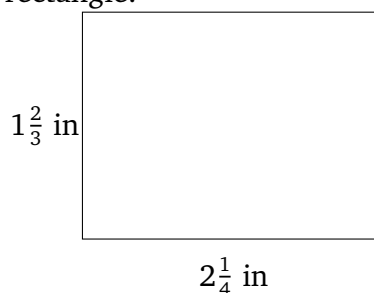


FIGURE 3.12: Find the area of the rectangle

Solution To find the area of a rectangle, we multiply the base and the height:

$$\begin{aligned}
 & 2\frac{1}{4} \cdot 1\frac{2}{3} \\
 &= \frac{9}{4} \cdot \frac{5}{3} \\
 &= \frac{9 \div 3}{4} \cdot \frac{5}{3 \div 3} \\
 &= \frac{3}{4} \cdot \frac{5}{1} \\
 &= \frac{15}{4} \\
 &= 3\frac{3}{4}
 \end{aligned}$$

Conclusion: The area of the rectangle is $3\frac{3}{4}$ square inches (or in^2). ■

Example 3.7.6 Find the area of the following triangle.

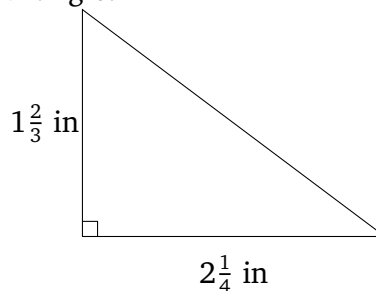


FIGURE 3.13: Find the area of the triangle

Solution Notice that a triangle is actually half a rectangle. See the following figure:

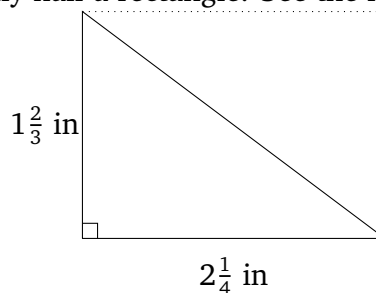


FIGURE 3.14: A triangle is half of a rectangle

To find the area of a triangle, we use the following formula:

$$\text{triangle area} = (\text{base})(\text{height}) \div 2$$

Now that we learned fraction, we could re-write the formula as:

$$\text{triangle area} = \frac{1}{2}(\text{base})(\text{height})$$

We find a rectangle's area by multiplying the base and the height. In the formula to find a triangle's area, the $\frac{1}{2}$ is there because a triangle's area is half of a rectangle, by Figure 3.14.

Now we have:

$$\begin{aligned}\text{area} &= \frac{1}{2} \cdot 2\frac{1}{4} \cdot 1\frac{2}{3} \\ &= \frac{1}{2} \cdot \frac{9}{4} \cdot \frac{5}{3} \\ &= \frac{1}{2} \cdot \frac{9 \div 3}{4} \cdot \frac{5}{3 \div 3} \\ &= \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{1} \\ &= \frac{15}{8} \\ &= 1\frac{7}{8}\end{aligned}$$

Conclusion: The area of the rectangle is $1\frac{7}{8}$ square inches (or in^2). ■