3.6 Add/Subtract Mixed Numbers

In this lesson, we will learn how to add and subtract mixed numbers. We still need the skill of finding common denominators we learned earlier.

3.6.1 Add/Subtract a Mixed Number and an Integer

The following two examples should be easy to understand.

Example 3.6.1

$$5\frac{2}{7}+3=8\frac{2}{7}$$
$$5\frac{2}{7}-3=2\frac{2}{7}$$

The next example takes some thinking: When we subtract a mixed number like $5\frac{2}{7}$, it's equivalent to first subtracting 5 whole pies, and then subtracting $\frac{2}{7}$ of a pie. Here are the first few steps of doing $7-5\frac{2}{7}$:

$$\begin{aligned} & 7-5\frac{2}{7} \\ =\ & 7-5-\frac{2}{7} \\ =\ & 2-\frac{2}{7} \\ =\ & \ldots \end{aligned}$$

Next, think about the situation: There are 2 whole pies, and someone ate $\frac{2}{7}$ of one pie. There is still one whole pie left. The other pie, cut into 7 pieces with 2 pieces eaten, still has 5 pieces left. So we have: $2-\frac{2}{7}=1\frac{5}{7}$. The full solution is:

$$\begin{aligned} & 7-5\frac{2}{7} \\ =\ & 7-5-\frac{2}{7} \\ =\ & 2-\frac{2}{7} \\ =\ & 1\frac{5}{7} \end{aligned}$$

Let's look at another example:

Example 3.6.2

$$\begin{aligned} & 10-4\frac{9}{20} \\ =\ & 10-4-\frac{9}{20} \\ =\ & 6-\frac{9}{20} \\ =\ & 5\frac{11}{20} \end{aligned}$$

It's more complicated when negative numbers are involved. See the next two examples.

Example 3.6.3

$$\begin{aligned} & 4-10\frac{9}{20} \\ =\ & 4-10-\frac{9}{20} \\ =\ & -6-\frac{9}{20} \\ =\ & -6\frac{9}{20} \end{aligned}$$

If you have trouble understanding the last step, think about $-1-2=-3$ (we need to add 1 and 2).

Example 3.6.4

$$\begin{aligned} & -4-10\frac{9}{20} \\ =\ & -4-10-\frac{9}{20} \\ =\ & -14-\frac{9}{20} \\ =\ & -14\frac{9}{20} \end{aligned}$$

3.6.2 Add/Subtract Mixed Numbers with the Same Denominator

The key is to break a mixed number into an integer and a fraction. Let's look at a few examples.

Example 3.6.5

$$\begin{aligned} & 2\frac{1}{6}+3\frac{1}{6} \\ =\ & 2+\frac{1}{6}+3+\frac{1}{6} \\ =\ & 2+3+\frac{1}{6}+\frac{1}{6} \\ =\ & 5+\frac{1+1}{6} \\ =\ & 5+\frac{2}{6} \\ =\ & 5\frac{1}{3} \end{aligned}$$

The next example is more complicated.

Example 3.6.6

$$\begin{aligned} & 2\frac{5}{6}+3\frac{5}{6} \\ &= 2+\frac{5}{6}+3+\frac{5}{6} \\ &= 2+3+\frac{5}{6}+\frac{5}{6} \\ &= 5+\frac{5+5}{6} \\ &= 5+\frac{10}{6} \\ &= 5+\frac{5}{3} \\ &= 5+1\frac{2}{3} \\ &= 6\frac{2}{3} \end{aligned}$$

In this example, we changed $\frac{5}{3}$ to $1\frac{2}{3}$.
The next example shows how to do mixed number subtraction. Again, the key is to break the mixed number into an integer and a fraction.

Example 3.6.7

$$\begin{aligned} & 7\frac{5}{6}-4\frac{1}{6} \\ &= 7+\frac{5}{6}-4-\frac{1}{6} \\ &= 7-4+\frac{5}{6}-\frac{1}{6} \\ &= 3+\frac{5-1}{6} \\ &= 3+\frac{4}{6} \\ &= 3\frac{2}{3} \end{aligned}$$

The next example is more challenging. We need to review how to do subtraction like $31-17$. Once we line up those two numbers, we have:

$$\begin{array}{r} 31 \\ \underline{-17} \end{array}$$

Since we cannot do $1-7$ in the ones place, we use the concept of "borrowing" by taking 10 from 30, and put the 10 to the 1 in the ones place. Now, the number 31 is broken into 20 and 11.
Now we can do $20-10$ in the tens place, and $11-7$ in the ones place, and the final answer is 14.
We will use the same concept to do the following mixed number subtraction problem:

Example 3.6.8

$$
\begin{aligned}
& 7\frac{1}{6}-4\frac{5}{6} \\
&= 7+\frac{1}{6}-4-\frac{5}{6} \\
&= 7-4+\frac{1}{6}-\frac{5}{6} \\
&= 3+\frac{1}{6}-\frac{5}{6} \\
&= 2+1+\frac{1}{6}-\frac{5}{6} && \text{borrowing 1 from 3} \\
&= 2+\frac{6}{6}+\frac{1}{6}-\frac{5}{6} && \text{change 1 to } \frac{6}{6} \\
&= 2+\frac{7}{6}-\frac{5}{6} \\
&= 2+\frac{2}{6} \\
&= 2\frac{1}{3} && \text{reduce fraction}
\end{aligned}
$$

In this example, since we cannot subtract $\frac{5}{6}$ from $\frac{1}{6}$, we "borrowed" 1 from the integer 3, changed 1 to $\frac{6}{6}$, and then changed $\frac{1}{6}$ to $\frac{6}{6}+\frac{1}{6}=\frac{7}{6}$. Now we can subtract $\frac{5}{6}$ from $\frac{7}{6}$. This is the same strategy we used when we do $31-17=14$.
Things become more complicated when negative numbers are involved. See the next few examples.

Example 3.6.9

$$
\begin{aligned}
& -7\frac{1}{6}-4\frac{5}{6} \\
&= -7-\frac{1}{6}-4-\frac{5}{6} \\
&= -7-4-\frac{1}{6}-\frac{5}{6} \\
&= -11+(-\frac{1}{6})+(-\frac{5}{6}) \\
&= -11+\frac{(-1)+(-5)}{6} \\
&= -11+\frac{-6}{6} \\
&= -11+(-1) \\
&= -12
\end{aligned}
$$

To make it clear, we changed $-\frac{1}{6}-\frac{5}{6}$ to $(-\frac{1}{6})+(-\frac{5}{6})$. This is like changing $-1-2$ to $(-1)+(-2)$.

Example 3.6.10

$$\begin{aligned}
&-7\frac{5}{6}+4\frac{1}{6}\\
&=-7-\frac{5}{6}+4+\frac{1}{6}\\
&=-7+4-\frac{5}{6}+\frac{1}{6}\\
&=-3+\frac{-5}{6}+\frac{1}{6}\\
&=-3+\frac{-5+1}{6}\\
&=-3+\frac{-4}{6}\\
&=-3+\frac{-2}{3}\\
&=-3\frac{2}{3}
\end{aligned}$$

The last step takes some thinking. Think of $(-1)+(-2)=-3$. When we add two negative numbers, we actually add up the absolute value of those two numbers. This is why when we do $-3+\frac{-2}{3}$, we need to do $3+\frac{2}{3}=3\frac{2}{3}$, and then make it negative.

Example 3.6.11

$$\begin{aligned}
&1\frac{5}{6}-4\\
&=1+\frac{5}{6}-4\\
&=1-4+\frac{5}{6}\\
&=-3+\frac{5}{6}\\
&=-2-1+\frac{5}{6}\\
&=-2-\frac{6}{6}+\frac{5}{6}\\
&=-2+\frac{-6+5}{6}\\
&=-2+\frac{-1}{6}\\
&=-2\frac{1}{6}
\end{aligned}$$

Adding/subtracting mixed numbers with different denominators is more complicated, but the strategies are the same.

3.6.3 Summary

When we do mixed number addition/subtraction, we usually break up the mixed number into its integer part and fraction part. For example:

$$2\frac{3}{5}=2+\frac{3}{5}$$

$$-2\frac{3}{5} = -2 - \frac{3}{5}$$

A lot of practice is needed for mixed number addition/subtraction. Make sure you understand each step as you go.