3.5 Introduction to Mixed Number

In this lesson, we will learn the definition of mixed numbers, and how to convert between improper fraction (like $\frac{3}{2}$) and mixed number (like $1\frac{1}{2}$).

3.5.1 Definition of Mixed Number

Let's review the definition of fractions, and use $\frac{2}{3}$ as an example. Remember, we should look at the denominator, 3, first:

1. We cut the whole evenly into 3 pieces.
2. We take 2 of those pieces.

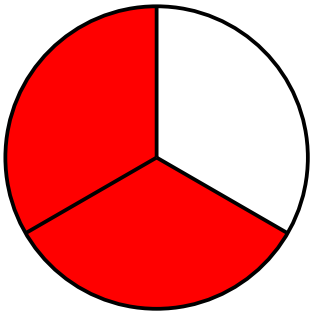

FIGURE 3.8: Red pieces in the pie represent $\frac{2}{3}$

Let's use the same concept to draw $\frac{7}{3}$.

1. We cut the whole evenly into 3 pieces.
2. We take 7 of those pieces. Since one pie only has 3 pieces, we need to cut more than one pie to have 7 pieces.

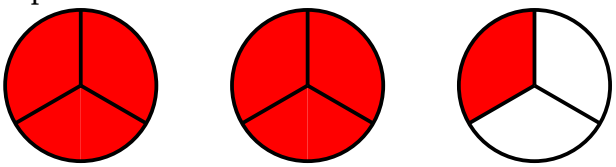

FIGURE 3.9: Red pieces represent $\frac{7}{3}$

If the numerator is bigger than the denominator, like in $\frac{7}{3}$, this fraction is called an *improper fraction*.

Another way to write $\frac{7}{3}$ is $2\frac{1}{3}$, representing 2 whole pies plus $\frac{1}{3}$ of a pie. A fraction like $2\frac{1}{3}$ is called a *mixed number*.

In other words, each mixed number can be converted to an improper fraction, and vice versa:

$$\frac{7}{3} = 2\frac{1}{3}$$

3.5.2 Convert between Improper Fractions and Mixed Numbers

Let's look at one more example by graphing. Write a mixed number and an improper fraction for the following graph:

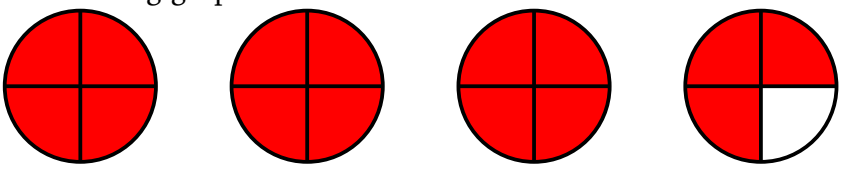

FIGURE 3.10: A graph representing a mixed number

In improper fraction format, the graph represents $\frac{15}{4}$, because there are 15 pieces of a quarter.

In mixed number format, the graph represents $3\frac{3}{4}$.
This implies:

$$\frac{15}{4} = 3\frac{3}{4}$$

Let's look at both graphs and both conversions:

$$\frac{7}{3} = 2\frac{1}{3}$$
$$\frac{15}{4} = 3\frac{3}{4}$$

The fraction $\frac{7}{3}$ has two whole pies because 3 goes into 7 twice. The fraction part of $2\frac{1}{3}$ is $\frac{1}{3}$ because there is one extra piece, or $7 \div 3 = 2R1$ (remainder is 1).
The fraction $\frac{15}{4}$ has three whole pies because 4 goes into 15 three times. The fraction part of $3\frac{3}{4}$ is $\frac{3}{4}$ because there are three extra pieces, or $15 \div 4 = 3R3$ (remainder is 3).
Once the above explanation makes sense, we can convert between improper fractions and mixed numbers without graphing.

Example 3.5.1 Convert $\frac{19}{7}$ to a mixed number.

Solution Since $19 \div 7 = 2R5$, it implies we can draw two whole pies, with each pie having 7 pieces. There are still 5 extra pieces, so we have:

$$\frac{19}{7} = 2\frac{5}{7}$$

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Example 3.5.2 Convert $4\frac{3}{5}$ to a mixed number.

Solution The mixed number's fraction part is $\frac{3}{5}$, implying each pie is cut into 5 pieces.
Since there are 4 whole pies, once each pie is cut into 5 pieces, there are a total of $4 \cdot 5 = 20$ pieces.
The fraction part is $\frac{3}{5}$, implying there are 3 extra pieces. Altogether, there are $4 \cdot 5 + 3 = 23$ pieces of one fifth of a pie. So we have:

$$4\frac{3}{5} = \frac{4 \cdot 5 + 3}{5} = \frac{23}{5}$$

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3.5.3 Mixed Numbers on Number Line

The following figures show how to locate mixed numbers on the number line. These figures are pretty self-explanatory. We need to count each unit (like from 0 to 1) is cut into how many segments.

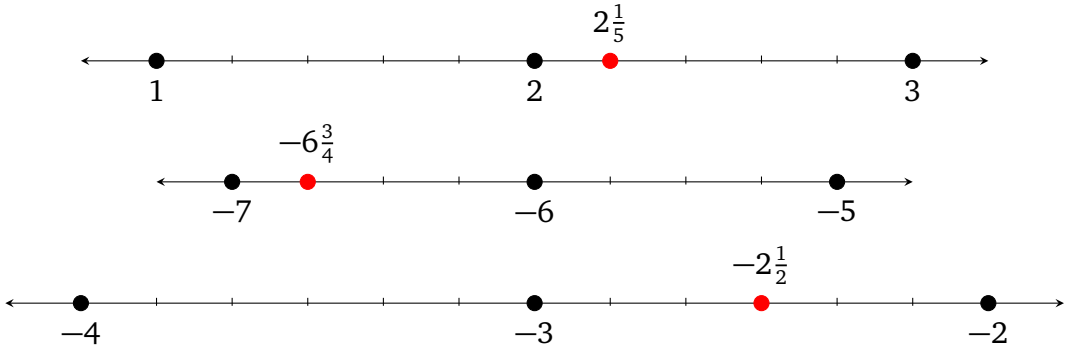

FIGURE 3.11: Locate mixed numbers on number line

On the last number line, note that the segment from -4 to -3 is cut evenly into 6 pieces, implying each piece represents $\frac{1}{6}$.
The red dot represents $-2\frac{3}{6}$. We must reduce the fraction: $-2\frac{3}{6} = -2\frac{1}{2}$.

3.5.4 Summary

Let's review what we learned in this lesson:

- To change a mixed number to an improper fraction, we do:

$$3\frac{4}{5} = \frac{3 \cdot 5 + 4}{5} = \frac{19}{5}$$

- To change an improper fraction to a mixed number, we first do a division:

$$19 \div 5 = 3R4$$

Next we have:

$$\frac{19}{5} = 3\frac{4}{5}$$

Understand that the 3 represents 3 whole pies, and the 4 represents 4 extra pieces (the remainder).