

## 3.4 Divide Fractions

In this lesson, we will learn how to divide fractions.

### 3.4.1 Understand Dividing Fractions

How many quarters are there in \$3.00? This is a division problem:

$$3 \div \frac{1}{4}$$

To do this problem, we divide all \$3.00 into quarters, and we will have  $3 \cdot 4 = 12$  quarters. So the solution is  $3 \div \frac{1}{4} = 12$ .

Next, let's make this example more interesting:

$$3 \div \frac{3}{4}$$

We need to find how many "three quarters" are in \$3.00? We will do this in two steps:

1. We divide 3 into quarters, and we have  $3 \cdot 4 = 12$  quarters.
2. We divide 12 quarters into groups of 3, and we have  $12 \div 3 = 4$  groups.

The solution is:

$$3 \div \frac{3}{4} = 3 \cdot 4 \div 3 = 12 \div 3 = 4$$

Recall that the fraction line is the same as the division symbol. We can re-write the solution as:

$$3 \div \frac{3}{4} = 3 \cdot \frac{4}{3} = \frac{3}{1} \cdot \frac{4}{3} = \frac{12}{3} = 4$$

The second way is how most middle school teachers teach dividing fractions: We change division to multiplication, and at the same time "flip" the second fraction. Unfortunately, the procedure doesn't make sense. I hope the money model helps you understand why the procedure makes sense.

I have to say it's rather difficult to understand fraction division. If you are not interested in understanding why, simply remember the rule.

#### Example 3.4.1

$$\begin{aligned} & \frac{2}{3} \div \frac{2}{9} \\ &= \frac{2}{3} \cdot \frac{9}{2} \\ &= \frac{2 \div 2}{3 \div 3} \cdot \frac{9 \div 2}{2 \div 2} \\ &= \frac{1}{1} \cdot \frac{3}{1} \\ &= \frac{3}{1} \\ &= 3 \end{aligned}$$

Never let fractions like  $\frac{3}{1}$  be the final answer! Change it to an integer.

### 3.4.2 Dividing Fractions involving Integers

If an integer is involved in fraction division, we first change the integer to a fraction by adding 1 to the denominator. This is similar to the rule for fraction multiplication.

#### Example 3.4.2

$$\begin{aligned}
 & 10 \div \frac{5}{9} \\
 &= \frac{10}{1} \div \frac{5}{9} \\
 &= \frac{10}{1} \cdot \frac{9}{5} \\
 &= \frac{10 \div 5}{1} \cdot \frac{9}{5 \div 5} \\
 &= \frac{2}{1} \cdot \frac{9}{1} \\
 &= \frac{18}{1} \\
 &= 18
 \end{aligned}$$

#### Example 3.4.3

$$\begin{aligned}
 & \frac{5}{9} \div 10 \\
 &= \frac{5}{9} \div \frac{10}{1} \\
 &= \frac{5}{9} \cdot \frac{1}{10} \\
 &= \frac{5 \div 5}{9} \cdot \frac{1}{10 \div 5} \\
 &= \frac{1}{9} \cdot \frac{1}{2} \\
 &= \frac{1}{18}
 \end{aligned}$$

### 3.4.3 Fraction Division Word Problems

**Example 3.4.4** A school won a grant, and will spend  $\frac{2}{3}$  of it to purchase textbooks. Six committees will evenly share the textbook fund to purchase textbooks of 6 subjects. What fraction of the total grant will be used by each committee to purchase textbooks?

**Solution** In this problem,  $\frac{2}{3}$  will be evenly divided into 6 pieces, a division problem:

$$\begin{aligned}
 & \frac{2}{3} \div 6 \\
 &= \frac{2}{3} \div \frac{6}{1} \\
 &= \frac{2}{3} \cdot \frac{1}{6} \\
 &= \frac{2 \div 2}{3} \cdot \frac{1}{6 \div 2} \\
 &= \frac{1}{3} \cdot \frac{1}{3} \\
 &= \frac{1}{9}
 \end{aligned}$$

**Conclusion:** Each committee will use  $\frac{1}{9}$  of the total grant to purchase textbooks. ■

**Example 3.4.5** Jack will use his car to move a pile of rocks which weigh  $\frac{7}{8}$  of a ton. Jack's car can load only  $\frac{1}{16}$  of a ton. How many trips will it take to move the whole pile?

**Solution** In this problem, we will repeatedly take away  $\frac{1}{16}$  of a ton from  $\frac{7}{8}$  of a ton, a division problem:

$$\begin{aligned} & \frac{7}{8} \div \frac{1}{16} \\ &= \frac{7}{8} \cdot \frac{16}{1} \\ &= \frac{7}{8 \div 8} \cdot \frac{16 \div 8}{1} \\ &= \frac{7}{1} \cdot \frac{2}{1} \\ &= \frac{14}{1} \\ &= 14 \end{aligned}$$

**Conclusion:** It will take 14 trips to move the whole pile of rocks. ■

### 3.4.4 Summary

Let's review what we learned in this lesson:

- To divide two fractions, we change division to multiplication, and at the same time, flip the second fraction. For example:

$$\frac{1}{3} \div \frac{1}{2} = \frac{1}{3} \cdot \frac{2}{1} = \frac{2}{3}$$

- If an integer is involved in fraction division, we first change the integer to a fraction, and then carry out the procedures above. For example:

$$\frac{1}{3} \div 2 = \frac{1}{3} \div \frac{2}{1} = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$$