

3.3 Multiply Fractions

In this lesson, we will learn how to multiply fractions.

3.3.1 Multiply Two Fractions

We will start with a very important concept: The English word "of," in many cases, can be translated into the multiplication sign in math. For example, "twice of 5" can be translated into $2 \cdot 5$.

Similarly, "half of half a dollar (50 cents)" can be translated into $\frac{1}{2} \cdot \frac{1}{2}$. We know the answer should be 25 cents, or $\frac{1}{4}$. We have:

$$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

We can see how to multiply fractions: We simply multiply the numerators and denominators. This is much easier than adding fractions—There is no need to find the common denominator when we multiply fractions.

Example 3.3.1

$$\begin{aligned} & \frac{2}{3} \cdot \frac{2}{5} \\ &= \frac{2 \cdot 2}{3 \cdot 5} \\ &= \frac{4}{15} \end{aligned}$$

Remember: We must reduce fraction if we can. See the next example.

Example 3.3.2

$$\begin{aligned} & \frac{2}{3} \cdot \frac{3}{5} \\ &= \frac{2 \cdot 3}{3 \cdot 5} \\ &= \frac{6}{15} \\ &= \frac{6 \div 3}{15 \div 3} \\ &= \frac{2}{5} \end{aligned}$$

In the example above, we could reduce fractions before we multiply across:

$$\begin{aligned} & \frac{2}{3} \cdot \frac{3}{5} \\ &= \frac{2}{3 \div 3} \cdot \frac{3 \div 3}{5} \\ &= \frac{2}{1} \cdot \frac{1}{5} \\ &= \frac{2 \cdot 1}{1 \cdot 5} \\ &= \frac{2}{5} \end{aligned}$$

This skill will save us a lot of time if numbers are big. See the next example.

Example 3.3.3

$$\begin{aligned}
 & \frac{4}{5} \cdot \frac{3}{8} \cdot \frac{5}{9} \\
 &= \frac{4 \div 4}{5 \div 5} \cdot \frac{3 \div 3}{8 \div 4} \cdot \frac{5 \div 5}{9 \div 3} \\
 &= \frac{1}{1} \cdot \frac{1}{2} \cdot \frac{1}{3} \\
 &= \frac{1 \cdot 1 \cdot 1}{1 \cdot 2 \cdot 3} \\
 &= \frac{1}{6}
 \end{aligned}$$

In this example, if we don't reduce fractions first, we have to deal with big numbers like $4 \cdot 3 \cdot 5 = 60$ and $5 \cdot 8 \cdot 9 = 360$.

Here is a common mistake:

$$\begin{aligned}
 & \frac{2}{3} \cdot \frac{2}{5} \\
 &= \frac{2 \div 2}{3} \cdot \frac{2 \div 2}{5} \\
 &= \frac{1}{3} \cdot \frac{1}{5} \\
 &= \frac{1}{15}
 \end{aligned}$$

This is incorrect. When we reduce a fraction, we must divide a number in both the numerator and denominator. We cannot divide a number in two numerators, as in the example above. The correct answer is $\frac{2}{3} \cdot \frac{2}{5} = \frac{4}{15}$.

3.3.2 Multiply a Fraction and an Integer

When we multiply a fraction and an integer, we need to change the integer to a fraction, and then multiply two fractions.

Example 3.3.4

$$\begin{aligned}
 & \frac{2}{9} \cdot 2 \\
 &= \frac{2}{9} \cdot \frac{2}{1} \\
 &= \frac{2 \cdot 2}{9 \cdot 1} \\
 &= \frac{4}{9}
 \end{aligned}$$

We can change 2 into $\frac{2}{1}$ because $2 \div 1 = 2$.

We need to learn an important shortcut, which will save you tons of time later. Recall that the fraction line is the same as the division symbol. We can do:

$$\frac{3}{5} \cdot 10 = 10 \div 5 \cdot 3 = 2 \cdot 3 = 6$$

Let's look at a few more examples:

$$\begin{aligned}\frac{2}{3} \cdot 3 &= 3 \div 3 \cdot 2 = 1 \cdot 2 = 2 \\ \frac{2}{3} \cdot 6 &= 6 \div 3 \cdot 2 = 2 \cdot 2 = 4 \\ \frac{2}{3} \cdot 9 &= 9 \div 3 \cdot 2 = 3 \cdot 2 = 6\end{aligned}$$

This shortcut works as long as the denominator goes into the integer evenly. If not, we have to change the integer to a fraction, and then do fraction multiplication like in example 3.3.4.

3.3.3 Fraction Multiplication Word Problems

Example 3.3.5 A school won a \$5,000 grant, and will use $\frac{3}{4}$ of the grant to purchase graphing calculators. How much money will be used to purchase graphing calculators?

Solution This problem can be boiled down to this question: What is $\frac{3}{4}$ of 5,000? Again, the word "of" can be translated into the multiplication sign in math. We have:

$$\begin{aligned}\frac{3}{4} \cdot 5000 \\ &= 5000 \div 4 \cdot 3 \\ &= 1250 \cdot 3 \\ &= 3750\end{aligned}$$

We can use the shortcut to do fraction multiplication because the denominator 4 goes into 5000 evenly.

Conclusion: The school will spend \$3,750 to purchase graphing calculators. ■

Example 3.3.6 A school won a grant, and will evenly share the grant among 8 classes. In one class, Mr. Smith will use $\frac{2}{3}$ of the money for the class to purchase books. Mr. Smith will use what fraction of the school's grant to purchase books?

Solution Since the grant is evenly shared by 8 classes, each class gets $\frac{1}{8}$ of the grant. Now this problem can be boiled down to this question: What is $\frac{2}{3}$ of $\frac{1}{8}$? Again, the word "of" can be translated into the multiplication sign in math. We have:

$$\begin{aligned}\frac{2}{3} \cdot \frac{1}{8} \\ &= \frac{2 \div 2}{3} \cdot \frac{1}{8 \div 2} \\ &= \frac{1}{3} \cdot \frac{1}{4} \\ &= \frac{1 \cdot 1}{3 \cdot 4} \\ &= \frac{1}{12}\end{aligned}$$

Conclusion: Mr. Smith will use $\frac{1}{12}$ of the school's grant to purchase books. ■

3.3.4 Summary

Let's review what we learned in this lesson:

- To multiply two fractions, we simply multiply the numerators and denominators. For example:

$$\frac{1}{2} \cdot \frac{1}{3} = \frac{1 \cdot 1}{2 \cdot 3} = \frac{1}{6}$$

- When we multiply fractions, if we can reduce fractions before multiplying the numerators and denominators, we should reduce fractions first. This will avoid dealing with big numbers. For example:

$$\frac{4}{7} \cdot \frac{3}{8} = \frac{4 \div 4}{7} \cdot \frac{3}{8 \div 4} = \frac{1}{7} \cdot \frac{3}{2} = \frac{3}{14}$$

- To multiply a fraction and an integer, we first change the integer to a fraction, and then multiply the numerators and denominators. For example:

$$\frac{1}{3} \cdot 2 = \frac{1}{3} \cdot \frac{2}{1} = \frac{2}{3}$$

- When we multiply a fraction and an integer, if the denominator can go into the integer evenly, there is a shortcut:

$$\frac{2}{3} \cdot 6 = 6 \div 3 \cdot 2 = 2 \cdot 2 = 4$$