2.2 Add and Subtract Integers

In this lesson, we will learn how to add and subtract positive and negative integers.

2.2.1 Add Integers

I will show two methods to add integers. You choose the method which makes more sense for you. We will calculate $(-1)+(-2)$.

Method 1: The first method is the traditional number line method. Recall that the right side is the positive direction, and the left side is negative direction.

Step 1: Locate the first number, -1, on the number line.

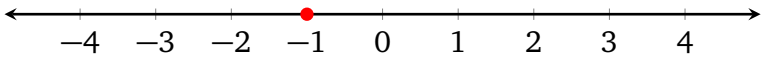

FIGURE 2.4: Locate -1 on the number line

Step 2: The second number is -2, meaning we will move to the *left* (negative direction) by two units.

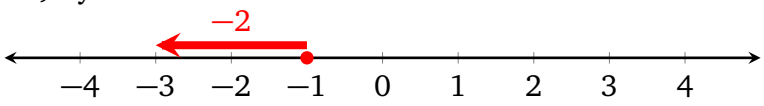

FIGURE 2.5: From -1 move to the left by 2 units

Step 3: After the move, we reached the number -3 on the number line. This implies that

$$(-1)+(-2)=-3$$

Method 2: The second method uses a money model. We deal with money every day, so most students easily understand this method.

Step 1: Say you are gambling. The first number is -1, meaning you lost \$1 in the first game.

Step 2: The second number is -2, meaning you lost \$2 in the second game.

Step 3: Since you lost in both games, altogether, you lost. This implies the answer must be negative.

Step 4: Since you lost in both games, all together, you lost \$1+\$2=\$3.

Finally, we have

$$(-1)+(-2)=-3$$

Let's look at a few more examples.

Example 2.2.1 Calculate $4+(-5)$

Solution We will use the number line method.

Step 1: Locate the first number, 4, on the number line.

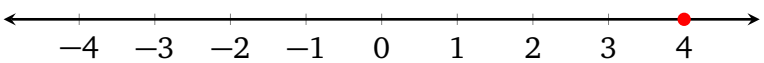

FIGURE 2.6: Locate 4 on the number line

Step 2: The second number is -5, meaning we will move to the *left* (negative direction) by five units.

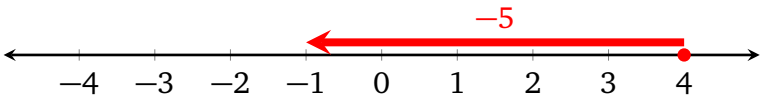

FIGURE 2.7: From 4 move to the left by 5 units

Step 3: After the move, we reached the number -1 on the number line. This implies:

$$4+(-5)=-1$$

■

Example 2.2.2 Calculate $(-4)+5$

Solution We will use the money model to solve this problem.

Step 1: Say you are gambling. The first number is -4, meaning you lost \$4 in the first game.

Step 2: The second number is 5, meaning you won \$5 in the second game.

Step 3: Since you won more money than you lost, altogether, you won. This implies the answer must be positive.

Step 4: Since you lost some and then won some, we should find the difference of those two numbers' absolute values: \$5−\$4=\$1.

Finally, we have

$$(-4)+5=1$$

■

If you are new to negative numbers, the number line method can help you understand integer operations. When numbers are big, it's difficult to locate numbers on the number line; the money model would work better.

Example 2.2.3 Calculate $(-44)+15$

Solution We will use the money model to solve this problem.

Step 1: Say you are gambling. The first number is -44, meaning you lost \$44 in the first game.

Step 2: The second number is 15, meaning you won \$15 in the second game.

Step 3: Since you lost more money than you won, altogether, you lost. This implies the answer must be negative.

Step 4: Since you lost some and then won some, we should find the difference of those two numbers' absolute values: \$44−\$15=\$29.

Finally, we have

$$(-44)+15=-29$$

Don't forget the negative sign in the answer (because you lost money). ■

2.2.2 Subtract a Positive Integer

Let's observe a pattern first:

$$3-2=1 \longleftrightarrow 3+(-2)=1$$
$$4-3=1 \longleftrightarrow 4+(-3)=1$$
$$1-3=-2 \longleftrightarrow 1+(-3)=-2$$

We can see the subtraction sign and negative symbol have the same functions! Starting today, it would be great if you can treat the subtraction sign as a negative symbol. When we subtract a positive number, we can treat it as "adding a negative number", and then use the methods we learned earlier to add integers.

Example 2.2.4 Calculate $-4-5$

Solution First, we change subtraction to "adding a negative":

$$-4-5=(-4)+(-5)$$

Note that the parentheses around -4 is optional, just to make it clear. The parentheses around -5 is needed, as it's confusing to write two symbols right next to each other, like $-4+-5$. Next, we will use the money model to solve this problem.

Step 1: Say you are gambling. The first number is -4, meaning you lost \$4 in the first game.

Step 2: The second number is -5, meaning you lost \$5 in the second game.

Step 3: Since you lost money in both games, altogether, you lost. This implies the answer must be negative.

Step 4: Since you lost in both games, we should find the sum of those two numbers' absolute values: \$5+\$4=\$9.

Finally, we have

$$-4-5=(-4)+(-5)=-9$$

■

Example 2.2.5 Calculate $4-5$

Solution First, we change subtraction to "adding a negative":

$$4-5=4+(-5)$$

Next, we will use the money model to solve this problem.

Step 1: Say you are gambling. The first number is 4, meaning you won \$4 in the first game.

Step 2: The second number is -5, meaning you lost \$5 in the second game.

Step 3: Since you lost more money than you won, altogether, you lost. This implies the answer must be negative.

Step 4: Since you won some money and then lost some, we should find the difference of those two numbers' absolute values: \$5−\$4=\$1.

Finally, we have

$$4-5=4+(-5)=-1$$

■

Don't be silly when you do problems like $9-5$. There is no need to use the number line or money model, as $9-5=4$. :)

2.2.3 Subtract a Negative Integer

Here is the bottom line: When two negative signs are right next to each other, we change these two negative signs to one positive sign, as in

$$1-(-2)=1+2$$

For now, memorize this as a rule. In the next lesson, we will understand why.
Look at the difference between these two problems:

$$\begin{aligned} &1-(-2) \\ &=1+2 \\ &=3 \end{aligned} \qquad\qquad \begin{aligned} &-1-2 \\ &=-3 \end{aligned}$$

We only change two negative signs to one positive sign if they are right next to each other. In $-1-2$, a number separated those two negative signs, so we cannot change them to one positive sign.

Example 2.2.6 Calculate $-4-(-5)$

Solution First, we change two negative signs to one positive sign:

$$-4-(-5)=-4+5$$

The rest of the solution is the same as in example 2.2.2. Finally, we have

$$-4-(-5)=-4+5=1$$

■

Finally, let's look at an example to put together what we learned in this lesson.

Example 2.2.7 Calculate $-3-(-5)-(-9)-14$

Solution The first step is to change two negative signs (right next to each other) to one positive sign:

$$\begin{aligned}&-3-(-5)-(-9)-14\\=&-3+5+9-14\end{aligned}$$

Next, we use either the number line model or money model to do additions and subtractions step by step. The full solution is:

$$\begin{aligned}&-3-(-5)-(-9)-14\\=&-3+5+9-14\\=&2+9-14\\=&11-14\\=&-3\end{aligned}$$

■