

## 1.7 Greatest Common Factor and Least Common Multiple

In this lesson, we learn how to find the GCF (Greatest Common Factor) and LCM (Least Common Multiple) of a group of numbers (usually two numbers).

For example, the GCF of 6 and 8 is 2. The LCM of 6 and 8 is 24. These two concepts will help you better understand numbers. In later lessons, we need to find LCM of denominators when we add or subtract fractions.

### 1.7.1 GCF (Greatest Common Factor)

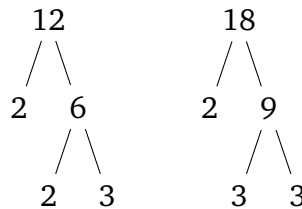
We will find the GCF of 12 and 18. We will list both numbers' factors:

- Factors of 12 are: 1, 2, 3, 4, 6, 12.
- Factors of 18 are: 1, 2, 3, 6, 9, 18.

The *common* factors of 12 and 18 are: 1, 2, 3, 6.

The GCF (Greatest Common Factor) of 12 and 18 is 6.

Note that we listed all factors of 12 and 18 in order to find their GCF. There is an easier method. We will prime factor both numbers:



We have:

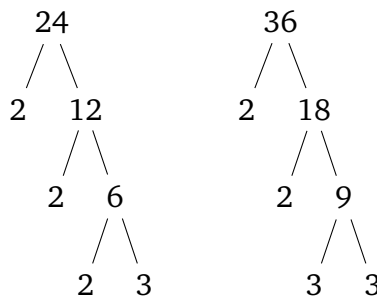
$$\begin{aligned}
 12 &= 2 \cdot 2 \cdot 3 \\
 18 &= 2 \cdot 3 \cdot 3
 \end{aligned}$$

Since 12 and 18 share the following factors, one 2 and one 3, the GCF of 12 and 18 is  $2 \cdot 3 = 6$ .

Let's look at another example.

**Example 1.7.1** Find the GCF of 24 and 36.

**Solution** We will prime factor both 24 and 36:



We have:

$$\begin{aligned}
 24 &= 2 \cdot 2 \cdot 2 \cdot 3 \\
 36 &= 2 \cdot 2 \cdot 3 \cdot 3
 \end{aligned}$$

We can see 24 and 36 share the following factors, two 2's and one 3, so the GCF of 24 and 36 is  $2 \cdot 2 \cdot 3 = 12$ . ■

**Example 1.7.2** Find the GCF of 8 and 15.

**Solution** Prime factor both numbers, we have:

$$8 = 2 \cdot 2 \cdot 2$$

$$15 = 3 \cdot 5$$

We can see 8 and 15 don't share any factors. The GCF of 8 and 15 is 1, because 1 is a factor of all natural numbers. ■

**Example 1.7.3** Find the GCF of 5 and 7.

**Solution** Both 5 and 7 are prime numbers, so it's impossible for them to share any factor except 1. So 1 is the GCF of 5 and 7. ■

**Example 1.7.4** Find the GCF of 10 and 20.

**Solution** Since 10 goes into 20 evenly, the GCF of 10 and 20 is simply 10, because 10 goes into both 10 and 20 evenly. ■

## 1.7.2 LCM (Least Common Multiple)

We will find the LCM of 6 and 8. First, we will find the first few multiples of 6 and 8:

- The first few multiples of 6 are: 6, 12, 18, 24, 30, 36, 42, 48, 54, ....
- The first few multiples of 8 are: 8, 16, 24, 32, 40, 48, 56....

We can see 6 and 8 have some *common* multiples: 24, 48, .... The *least* common multiple is 24.

Later, when we do  $\frac{1}{6} + \frac{1}{8}$ , we need to change the denominator of both fractions to 24, the LCM of 6 and 8. We will learn details later, but here is the solution:

$$\begin{aligned} & \frac{1}{6} + \frac{1}{8} \\ &= \frac{1 \cdot 4}{6 \cdot 4} + \frac{1 \cdot 3}{8 \cdot 3} \\ &= \frac{4}{24} + \frac{3}{24} \\ &= \frac{4 + 3}{24} \\ &= \frac{7}{24} \end{aligned}$$

As an example, we will find the LCM of 6 and 8 in two methods.

### Method 1 to Find LCM

We will prime factor both 6 and 8:

$$6 = 2 \cdot 3$$

$$8 = 2 \cdot 2 \cdot 2$$

The LCM of 6 and 8 must *cover* the prime factors of both numbers:

- The LCM of 6 and 8 must have three 2's as factors, as 6 needs one 2 and 8 needs three 2's.
- The LCM of 6 and 8 must have one 3 as a factor, as 6 needs one 3.

Putting them together, the LCM must have three 2's and one 3 as factors. We have:  $\text{LCM} = 2 \cdot 2 \cdot 2 \cdot 3 = 24$ .

To verify, we have  $\frac{24}{6} = 4$ , and  $\frac{24}{8} = 3$ .

### Method 2 to Find LCM

List the first few multiples of the *bigger* number until the smaller number goes into a multiple evenly:

The first few multiples of 8 are: 8, 16, 24.

We stopped here because 6 goes into 24 evenly, so 24 is the LCM of 6 and 8. To verify, we have  $\frac{24}{6} = 4$ , and  $\frac{24}{8} = 3$ .

Which method should we use? Most of the time, the second method is faster, so try the second method first. Sometimes the first method is faster.

**Example 1.7.5** Find the LCM of 4 and 12.

**Solution** Since 4 goes into 12, the LCM of 4 and 12 is simply 12.

To verify, we have  $\frac{12}{4} = 3$  and  $\frac{12}{12} = 1$ . ■

**Example 1.7.6** Find the LCM of 8 and 18.

**Solution** We list the first few multiples of 18, until 8 goes evenly into one of them:

Multiples of 18: 18, 36, 54, 72.

We stop here because  $\frac{72}{8} = 9$ . The LCM of 8 and 18 is 72.

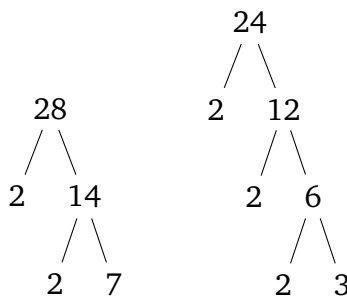
To verify, we have  $\frac{72}{8} = 9$  and  $\frac{72}{18} = 4$ . ■

**Example 1.7.7** Find the LCM of 24 and 28.

**Solution** We list the first few multiples of 28, until 24 goes evenly into one of them:

Multiples of 28: 28, 56, 84, 112, 140...

24 doesn't go evenly into any of them. After listing 5 multiples, we know this method is not the best method to find the LCM. We will try the other method. We will prime factor both numbers:



We have:

$$28 = 2 \cdot 2 \cdot 7$$

$$24 = 2 \cdot 2 \cdot 2 \cdot 3$$

The LCM of 28 and 24 must *cover* the prime factors of both numbers:

- The LCM must have three 2's as factors, as 24 needs three 2's and 28 needs two 2's.
- The LCM must have one 3 as a factor, as 24 needs one 3.
- The LCM must have one 7 as a factor, as 28 needs one 7.

Putting them together, the LCM must have three 2's, one 3 and one 7 as factors. We have:  
 $\text{LCM} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 7 = 168.$

To verify, we have  $\frac{168}{28} = 6$ , and  $\frac{168}{24} = 7$ . ■