1.6 Prime Number

Prime numbers are the backbone of all numbers. In this lesson, we will learn prime numbers and how to prime factor a number.

1.6.1 Factor and Prime

Since $\frac{8}{2} = 4$, we say 2 is a factor of 8, because 2 goes into 8 four times with no remainder.

Let's look at factors of the first few numbers, starting with 2:

- 2 has two factors: 1 and 2.

- 3 has two factors: 1 and 3.

- 4 has three factors: $1, 2$ and 4.

- 5 has two factors: 1 and 5.

- 6 has four factors: $1, 2, 3$ and 6. ...

We can see a number (except 1) has at least two factors: 1 and the number itself.

If a number has only two factors, 1 and the number itself, this number is a *prime number*. For example, 2, 3 and 5 are prime numbers.

If a number has more than two factors, this number is a *composite number*. For example, 4 and 6 are composite numbers.

If a number is an even number bigger than 2, it has at least 3 factors: 1, 2 and itself. All even numbers, except 2, are composite numbers.

It's useful to memorize the first few prime numbers, which will be used frequently later: $2, 3, 5, 7, 11, 13, \ldots$

Note that 1 is neither a prime number nor a composite number.

Example 1.6.1 Is 245 prime or composite?

Solution When we judge whether a number is prime or composite, we look at the first few prime numbers, $2, 3, 5, 7, 11, 13, \ldots$, and see whether any of them goes into the number evenly.

- 2 doesn't go into 245 evenly, because 245 is odd.

- 3 doesn't go into 245 evenly, because 3 doesn't go evenly into $2 + 4 + 5 = 11$.

- 5 goes into 245 evenly, because the last digit of 245 is 5.

We can tell 245 is composite because 5 goes into 245 evenly, implying 245 has at least 3 factors: 1, 5 and 245. ∎

Example 1.6.2 Is 199 prime or composite?

Solution When we judge whether a number is prime or composite, we look at the first few prime numbers, $2, 3, 5, 7, 11, 13, \ldots$, and see whether any of them goes into the number evenly.

- 2 doesn't go into 199 evenly, because 199 is odd.

- 3 doesn't go into 199 evenly, because 3 doesn't go evenly into $1 + 9 + 9 = 19$.

- 5 doesn't go into 199 evenly, because the last digit of 199 is not 0 or 5.

- To judge whether 7 goes into 199 evenly, it's reasonable to use a calculator. We can see $\frac{199}{7} = 28.42\ldots$, so 7 doesn't go into 199 evenly.

- With the help of calculator, we can tell neither 11 nor 13 goes into 199 evenly.

Theoretically, we must test all prime numbers smaller than 199: $17, 19, 23, \ldots$. In reality, no reasonable instructor would ask you to spend so much time to judge whether a number if prime or composite. After you test $2, 3, 5, 7$ and 11, and if none of them goes into the number evenly, it's reasonable to say the number is prime.

For this example, 199 is indeed a prime number.   ■

1.6.2 Prime Factoring

Prime numbers are backbones of numbers. Each composite number can be written as the product of prime numbers. For example, $15 = 3 \cdot 5$ and $12 = 2 \cdot 2 \cdot 3$. The last equation can be re-written in a simpler way as $12 = 2^2 \cdot 3$. We will learn how to prime factor numbers.

Example 1.6.3 Prime factor 12.

Solution First, write down the first few prime numbers: $2, 3, 5, 7, 11$. Rarely would an instructor expect you to use bigger prime numbers in prime factoring. We will prime factor 12 as an example.

To prime factor the number 12, we look at the first prime number, 2, and see whether 2 goes into 12. It does! We have $12 = 2 \cdot 6$.

In $2 \cdot 6$, the number 2 is prime, and we cannot further break it down; the number 6 is not prime, and can be further factored.

Again, we go back to the list of prime numbers, and see whether the first prime number 2 goes into 6. It does! Now we have $12 = 2 \cdot 6 = 2 \cdot 2 \cdot 3$. We can stop here because all numbers in the product $2 \cdot 2 \cdot 3$ are prime numbers.

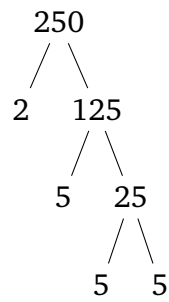We can re-write the product with exponent: $12 = 2^2 \cdot 3$.

It's easier to use a tree to prime factor numbers:

```
        12
       /  \
      2    6
          /  \
         2    3
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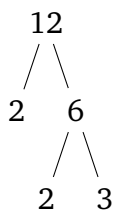
Note that we don't use the number 1 in prime factoring. Otherwise, we could keep going with no end like $12 = 2^2 \cdot 3 \cdot 1 \cdot 1 \cdot \ldots$.

Example 1.6.4 Prime factor 250.

Solution We will use a factor tree to solve this problem.

$$
\begin{array}{c}
250 \\
\diagup \ \diagdown \\
2 \quad 125 \\
\diagup \ \diagdown \\
5 \quad 25 \\
\diagup \ \diagdown \\
5 \quad 5
\end{array}
$$

The prime factor of 250 is $250 = 2 \cdot 5^3$. ∎

We can see a big number like 250 can be broken down to the product of very small prime numbers.