1.5 Divisibility Test

Number sense is critical in the study of mathematics. The first step of building number sense is to memorize the multiplication table. In this lesson, we will learn how to tell whether a number goes into another evenly. For example, without doing division, we know 2 goes into 126 evenly (meaning $\frac{126}{2}$ doesn't have remainder). This lesson is also important in building number sense.

1.5.1 Divisibility of 2

Let's look at the first few multiples of 2:

$$2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, \ldots$$

We can see they are all even numbers, so 2 goes into all even numbers evenly. In other words, 2 goes into a number evenly if its last digit is $0, 2, 4, 6$ or 8.

1.5.2 Divisibility of 10

Let's look at some multiples of 10:

$$10, 20, 30, 40, 50, \ldots, 100, 110, \ldots, 1230, 1240, \ldots$$

It's easy to see that 10 goes evenly into all numbers whose last digit is 0.

1.5.3 Divisibility of 5

Let's look at some multiples of 5:

$$5, 10, 15, 20, 25, 30, \ldots, 100, 105, 110, \ldots, 1230, 1235, \ldots$$

It's easy to see that 5 goes evenly into all numbers whose last digit is 0 or 5.

1.5.4 Divisibility of 3 and 9

Let's look at some multiples of 3:

$$3, 6, 9, 12, 15, \ldots, 99, 102, 105, \ldots, 300, 303, 306, 309, 312, \ldots 3333, 3336, 3339, 3342, \ldots$$

Let's add up all digits of some of these numbers:

$$15 : 1 + 5 = 6$$
$$102 : 1 + 0 + 2 = 3$$
$$309 : 3 + 0 + 9 = 12$$
$$3342 : 3 + 3 + 4 + 2 = 15$$

Notice that 3 goes evenly into all these sums: $6, 3, 12, 15$.

To judge whether 3 goes into a number evenly, we add up all digits of this number. If 3 goes into the sum evenly, 3 goes into the number evenly.

Example 1.5.1 Does 3 go into 195 and 1984 evenly?

Solution We add up all digits of 195, and we have $1 + 9 + 5 = 15$. Since 3 goes into 15 evenly, 3 must go into 195 evenly. We can use a calculator to verify this: $\frac{195}{3} = 65$.

We add up all digits of 1984, and we have $1 + 9 + 8 + 4 = 22$. Since 3 does not go into 22 evenly, 3 does not go into 1994 evenly. We can use a calculator to verify this: $\frac{1984}{3} = 661.333....$ ∎

This rule also works for 9, but doesn't work for any other numbers.

Example 1.5.2 Does 9 go into 396 and 987 evenly?

Solution We add up all digits of 396, and we have $3 + 9 + 6 = 18$. Since 9 goes into 18 evenly, 9 must go into 396 evenly. We can use a calculator to verify this: $\frac{396}{9} = 44$.

We add up all digits of 987, and we have $9 + 8 + 7 = 24$. Since 9 does not go into 24 evenly, 9 does not go into 987 evenly. We can use a calculator to verify this: $\frac{987}{9} = 109.666....$ ∎

Again, don't try to use this rule for 4, 5, or any number other than 3 and 9. Next, let's review all divisibility rules we learned.

Example 1.5.3 Decide whether 2, 3, 5, 9 and 10 go into $12,345$ evenly.

Solution
- Since $12,345$ is odd, 2 does not go into $12,345$ evenly.
- Add up all digits of $12,345$, we have $1 + 2 + 3 + 4 + 5 = 15$. Since 3 goes into 15 evenly, 3 must go into $12,345$ evenly.
- Since the last digit of $12,345$ is 5, 5 must go into $12,345$ evenly.
- Add up all digits of $12,345$, we have $1 + 2 + 3 + 4 + 5 = 15$. Since 9 does not go into 15 evenly, 9 does not go into $12,345$ evenly.
- Since the last digit of $12,345$ is 5, 10 does not go into $12,345$ evenly. ∎

Example 1.5.4 Decide whether 2, 3, 5, 9 and 10 go into $65,430$.

Solution
- Since $65,430$ is even, 2 goes into $65,430$ evenly.
- Add up all digits of $65,430$, we have $6 + 5 + 4 + 3 + 0 = 18$. Since 3 goes into 18 evenly, 3 must go into $65,430$ evenly.
- Since the last digit of $65,430$ is 0, 5 must go into $65,430$ evenly.
- Add up all digits of $65,430$, we have $6 + 5 + 4 + 3 + 0 = 18$. Since 9 goes into 18 evenly, 9 goes into $65,430$ evenly.
- Since the last digit of $65,430$ is 0, 10 goes into $65,430$ evenly. ∎

1.5.5 Summary

Let's review important concepts in this lesson.
- 2 goes into all even numbers (ending with $0, 2, 4, 6$ or 8).
- If 3 goes evenly into the sum of all digits of a number, 3 goes into the number evenly.
- 5 goes evenly into all numbers whose last digit is 5 or 10.
- If 9 goes evenly into the sum of all digits of a number, 9 goes into the number evenly.

- 10 goes evenly into all numbers whose last digit is 10.

Granted, if the question is: "Can 3 go into 123 evenly?", you could use the calculator to do the division $123 \div 3$ and see whether the quotient is an integer or decimal. However, the content of this lesson is very important in building your number sense, which is very important in later mathematics study.