1.4 Geometry Basics

In this lesson, we will learn how to find the perimeter and area of rectangles, squares and triangles.

1.4.1 Rectangle Perimeter

The perimeter of a rectangle is the total distance around the rectangle's edge. In other words, if you walk along a rectangle's edge once, the distance you will cover is the rectangle's perimeter.

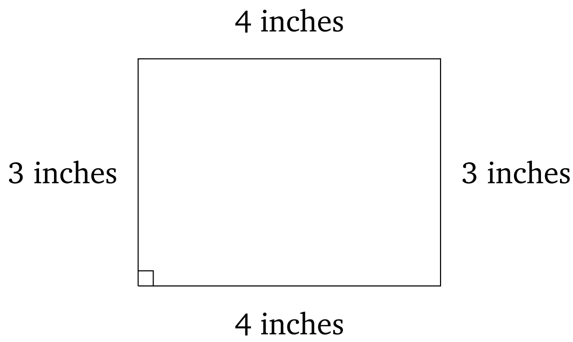

FIGURE 1.3: a 4-inch-by-3-inch rectangle

In this figure, we say the *base* of the rectangle is 4 inches, while the *height* of the rectangle is 3 inches.

A rectangle's base is also called *length*, and a rectangle's height is also called *width*.

This rectangle's perimeter is:

$$4+3+4+3=14 \text{ in}$$

In a rectangle, the opposite sides always have the same length. Instead of writing $4+4$, we could write $2\cdot 4$; instead of writing $3+3$, we could write $2\cdot 3$. So we could calculate a rectangle's perimeter this way:

$$2(4+3)=2\cdot 7=14 \text{ in}$$

Here is the formula for a rectangle's perimeter:

$$\text{rectangle perimeter} = 2(\text{base}+\text{height})$$

1.4.2 Square Perimeter

A square is a special rectangle. A rectangle's opposite sides have the same length, while all four sides of a square have the same length.

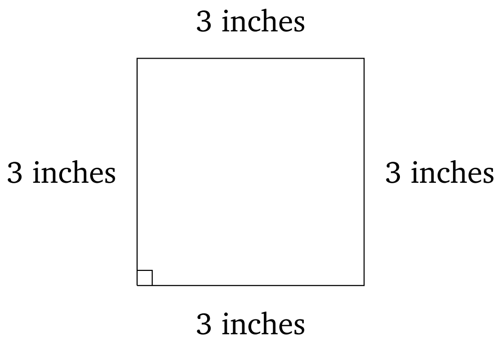

FIGURE 1.4: a 3-inch-by-3-inch square

To find the square's perimeter, we simply do:

$$\begin{aligned}&\text{square perimeter}\\&= 4 \cdot \text{base}\\&= 4 \cdot 3\\&= 12 \text{ in}\end{aligned}$$

1.4.3 Triangle Perimeter

There is no formula to find a triangle's perimeter—we simply add up the lengths of a triangle's three sides.

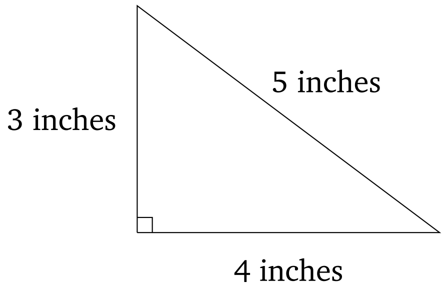

FIGURE 1.5: a triangle

This triangle's perimeter is:

$$\text{triangle perimeter} = 3 + 4 + 5 = 12 \text{ in}$$

1.4.4 Rectangle Area

Earlier, we learned how to find a figure's perimeter—it's the distance around the edge of the figure. Area is a different concept. Let's look at a rectangle cut into small "unit squares." Each unit square's base is 1 unit in length.

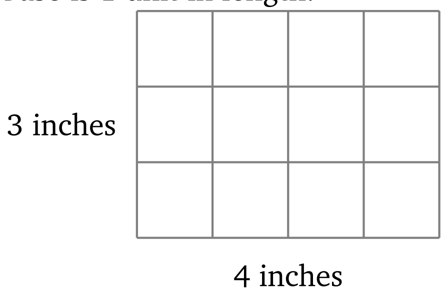

FIGURE 1.6: a 4-inch-by-3-inch rectangle

Each 1-inch-by-1-inch unit square has an area of 1 square inch. This rectangle has a total of $4 \cdot 3 = 12$ such unit squares, so the rectangle's area is 12 square inches.

The formula of a rectangle's area is:

$$\text{rectangle area} = \text{base} \cdot \text{height}$$

or

$$\text{rectangle area} = \text{length} \cdot \text{width}$$

Note that the unit of this rectangle is "square inch". This is different from the unit of a rectangle's perimeter—inch. The word "square" implies we are dealing with area, not length.

Instead of writing the words "square inches", we could also write in^2. Don't confuse this with "square of a number". Let's make a comparison:

- "3^2" means "three squared", or $3 \cdot 3 = 9$.
- "3 in^2" means "three square inches", the size of three 1-inch-by-1-inch unit squares.

1.4.5 Square Area

A square is a special rectangle, so it's area formula is the same as a rectangle's area formula, except a square's base and height always have the same length. Because of this, instead of writing a square's area formula as base · base, we can write a square's area formula as

$$\text{square area} = \text{base}^2$$

Example 1.4.1 Find the area of the following square.

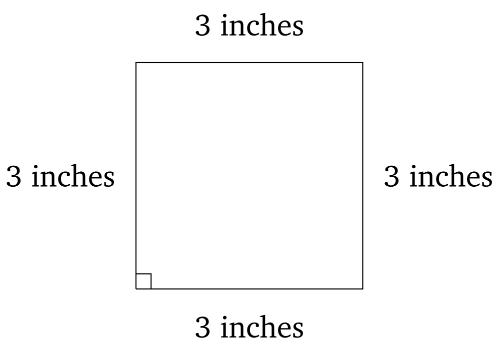

FIGURE 1.7: a 3-inch-by-3-inch square

Solution To find the square's area, we simply do:

$$\text{square area} = \text{base}^2 = 3^2 = 9 \text{ in}^2$$

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1.4.6 Triangle Area

In the following figure, we can clearly see a triangle is half as big as a rectangle with the same base and height.

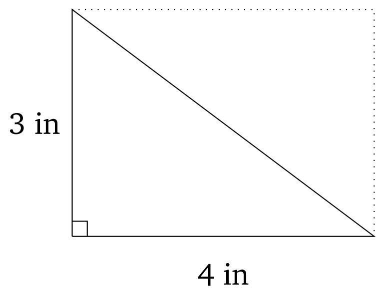

FIGURE 1.8: A triangle is half as big as a rectangle.

To find the area of a triangle, we first find the rectangle's area by (base) · (height), and then divide the rectangle's area by 2:

$$\text{triangle area} = \text{base} \cdot \text{height} \div 2$$

The triangle's area in Figure 1.8 is:

$$\text{triangle area} = 4 \cdot 3 \div 2 = 6 \text{ in}^2$$

In Figure 1.8, the triangle has a right angle (90 degrees). It is called a *right triangle*. The same area formula works for non-right triangles.

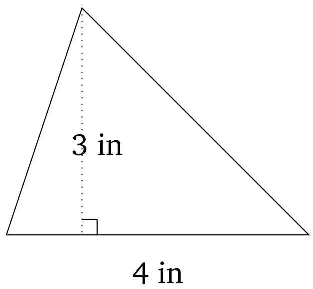

FIGURE 1.9: a non-right triangle

The triangle's area in Figure 1.9 is still:

$$\begin{aligned} &\text{triangle area} \\ &= \text{base} \cdot \text{height} \div 2 \\ &= 4 \cdot 3 \div 2 \\ &= 12 \div 2 \\ &= 6 \text{ in}^2 \end{aligned}$$

In Figure 1.9, the dotted line is the triangle's *height*. The small square at the bottom of the height implies 90 degrees. A triangle's height must form a 90-degree angle with the base.

1.4.7 Summary

Let's review important formulas we learned in this lesson:

- rectangle perimeter $= 2(\text{base} + \text{height})$
- square perimeter $= 4 \cdot \text{base}$

- rectangle area $=$ base $\cdot$ height
- square area $=$ base^2
- triangle area $=$ base $\cdot$ height $\div 2$