1.3 Division

We will review division in this lesson. Division is used when we divide a number evenly into certain groups. Look at this figure:

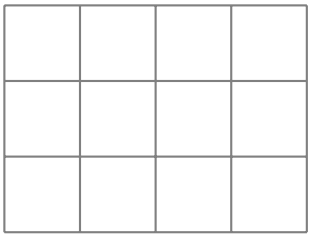

FIGURE 1.2: a 4 by 3 grid

There are a total of 12 blocks. If we divide them into 3 rows, each row would have $12 \div 3 = 4$ blocks.

If we divide 12 blocks into 4 columns, each column would have $12 \div 4 = 3$ blocks.

A second way to understand division is to "repeatedly take way." Assume there are 12 blocks. If 3 blocks are taken away each time, it will take $12 \div 3 = 4$ turns to take away all 12 blocks.

In $12 \div 3 = 4$, we call the result 4 the *quotient* of 12 and 3. Earlier, we learned the words *sum*, *difference* and *product*. Please memorize the meaning of these 4 words. For example, when if you see the word *product*, you know you are dealing with multiplication.

1.3.1 Multiplication and Division

Multiplication and division are *inverse operations*. For example:

- $12 \div 3 = 4$ as $3 \cdot 4 = 12$
- $15 \div 3 = 5$ as $3 \cdot 5 = 15$
- $0 \div 3 = 0$ as $3 \cdot 0 = 0$

This explains why we cannot "divide by 0". Let's look at:

$$3 \div 0$$

Assume we can do this, and we have

$$3 \div 0 = \text{some number}$$

Since multiplication and division are inverse operations, we have:

$$0 \cdot \text{some number} = 3$$

We know 0 times any number is always 0, so there is no way we can find a number which makes $0 \cdot \text{some number} = 3$ work. This is why we cannot divide a number by 0. Notice the difference:

$$0 \div 3 = 0 \text{ while } 3 \div 0 = \text{undefined}$$

If you do $3 \div 0$ on a calculator, you will receive an error.

1.3.2 Multiplication and Division Symbols

We are used to using the multiplication and division symbols we learned from grade school, as in $3 \times 4 = 12$ and $12 \div 3 = 4$.

Starting now, we will use $\cdot$ to represent multiplication, and use the fraction line to represent division. For example:

$$3 \cdot 4 = 12 \text{ and } \frac{12}{3} = 4$$

The fraction line simply means division. Once you understand this, you will understand fractions better.

1.3.3 Division Word Problems

Next, let's look at some examples where division is used. There are two situations where division is needed:

1. dividing a number evenly into groups, and
2. repeatedly taking away

Example 1.3.1 A teacher will do a math activity in a class. She will hand out 56 blocks to 8 students. If each student receives the same number of blocks, how many blocks will each student get?

Solution In this problem, we need to divide 56 blocks evenly into 8 groups. Each student will get $\frac{56}{8} = 7$ blocks. ■

Example 1.3.2 Omar bought 48 M&M candies, and plans to eat 4 of them every day. How many days will these candies last?

Solution In this problem, we need to repeatedly take away 4 candies from 48 candies. This is a division problem. These candies will last $\frac{48}{4} = 12$ days. ■

1.3.4 Remainder

Let's learn remainder in context.

Example 1.3.3 A teacher will do a math activity in a class. She will hand out 60 blocks to 8 students. If each student receives the same number of blocks, how many blocks will each student get?

Solution This example is very similar to example 1.3.1. However, we get a decimal quotient if we do $\frac{60}{8} = 7.5$. In this context, it's unlikely that the teacher will break up blocks and hand out half of a block to students. Each student will still get 7 blocks.

If each student gets 7 blocks, a total of $7 \cdot 8 = 56$ blocks will be handed out. The remainder is $60 - 56 = 4$, implying 4 blocks will be left.

Conclusion: Each student will get 7 blocks, with 4 blocks left. ■

Example 1.3.4 A teacher will do a math activity in a class. She will put all 60 blocks into containers. Each container can hold 8 blocks. How many containers will be needed?

Solution Compare this with example 1.3.3. We still do $\frac{60}{8} = 7.5$, but the context is different. The quotient is 7.5, implying we need 8 containers to hold 60 blocks, but the last container is not full.

If each container holds 8 blocks, 7 containers can hold a total of $7 \cdot 8 = 56$ blocks. The remainder is $60 - 56 = 4$, implying the last container is not full, holding 4 blocks.

Conclusion: 8 containers are needed, with the last container holding 4 blocks. ■

Note that we didn't use long-division to find remainders. Especially for big numbers, we will use a calculator, instead.

Example 1.3.5 Find the remainder of $\frac{121}{7}$ without doing long division.

Solution To find the remainder of $\frac{121}{7}$, the calculator tells us $\frac{121}{7} = 17.285714...$, implying the quotient is 17. This means, if we divide 121 blocks into groups of 7 blocks, there will be 17 groups, with some leftover. These 17 groups have $7 \cdot 17 = 119$ blocks, and the remainder is $121 - 119 = 2$. ■

We need this skill later to change an improper fraction to a mixed number, as in

$$\frac{121}{7} = 17\frac{2}{7}$$

1.3.5 Summary

Let's review important concepts in this lesson.

- Division is used when we divide a number evenly into groups, and when we repeatedly take away a smaller number from a bigger number.
- The result of division is called the quotient.
- Instead of using the division symbol $\div$, we can use the fraction line, as in $\frac{6}{2} = 3$.
- We can find the remainder without using long division, as in example 1.3.5.