Chapter 1: Logic and Sets

Student Outcomes for this Chapter

Section 1.1: The Language and Rules of Logic

Students will be able to:

- Identify propositions
- Compose and interpret the negation of a statement
- Use logical connectors (and/or) and conditional statements (if, then)
- Use truth tables to find truth values of basic and complex statements

Section 1.2: Sets and Venn Diagrams

Students will be able to:

- Use set notation and understand the null set
- Determine the universal set for a given context
- Use Venn diagrams and set notation to illustrate the intersection, union and complements of sets
- Illustrate disjoint sets, subsets and overlapping sets with diagrams
- Use Venn diagrams and problem-solving strategies to solve logic problems

Section 1.3: Describing and Critiquing Arguments.

Students will be able to:

- Understand the structure of logical arguments by identifying the premise(s) and conclusion.
- Distinguish between inductive and deductive arguments
- Make a set diagram to evaluate deductive arguments
- Determine whether a deductive argument is valid and/or sound

Section 1.4: Logical Fallacies.

Students will be able to:

- Identify common logical fallacies and their use in arguments

Section 1.1 The Language and Rules of Logic

Logic

Logic is the study of reasoning. Our goal in this chapter is to examine arguments to determine their validity and soundness. In this section we will look at propositions and logical connectors that are the building blocks of arguments. We will also use truth tables to help us examine complex statements.

Propositions

A **proposition** is a complete sentence that is either true or false. Opinions can be propositions, but questions or phrases cannot.

Example 1: Which of the following are propositions?

a. I am reading a math book.
b. Math is fun!
c. Do you like turtles?
d. My cat

The first and second items are propositions. The third one is a question and the fourth is a phrase, so they are not. We are not concerned right now about whether a statement is true or false. We will come back to that later when we examine full arguments.

Arguments are made of one or more propositions (called **premises**), along with a **conclusion**. Propositions may be **negated**, or combined with **connectors** like "and", and "or". Let's take a closer look at how these negations and logical connectors are used to create more complex statements.

Negation (not)

One way to change a proposition is to use its **negation**, or opposite meaning. We often use the word "not" to negate a statement.

Example 2: Write the negation of the following propositions.

a. I am reading a math book. Negation: I am not reading a math book.
b. Math is fun! Negation: Math is not fun!
c. The sky is not green. Negation: The sky is green (or not not green).
d. Cars have wheels Negation: Cars do not have wheels.

Multiple Negations

It is possible to use more than one negation in a statement. If you've ever said something like, "I can't not go," you are really saying you must go. It's a lot like multiplying two negative numbers which gives a positive result.

In the media and in ballot measures we often see multiple negations and it can be confusing to figure out what a statement means.

Example 3: Read the statement to determine the outcome of a yes vote.

"Vote for this measure to repeal the ban on plastic bags."

If you said that a yes vote would enable plastic bag usage, you are correct. The ban stopped plastic bag usage, so to repeal the ban would allow it again. This measure has a double negation and is also not very good for the environment.

Example 4: Read the statement to determine the outcome on mandatory minimum sentencing.

"The bill that overturned the ban on mandatory minimum sentencing was vetoed."

In this case mandatory minimum sentencing would not be allowed. The ban would stop it, and the bill to overturn it was vetoed. This is an example of a triple negation.

Logical Connectors (and, or)

When we use the word "and" between two propositions, it connects them to create a new statement that is also a proposition. For example, if you said, "when you go to the store, please get eggs ***and*** cereal," you would be expecting both items. For an *and* statement to be true, the connected propositions must both be true. If even one proposition is false (for instance, you get eggs but not cereal), the entire connected *and* statement is false.

The word "or" between two propositions similarly connects the propositions to create a new statement. In this case, if you said, "please get eggs ***or*** cereal," you would be expecting one or the other (but probably not both). For an *or* statement to be true, at least one of the propositions must be true (or both could be true).

Exclusive vs. Inclusive *or*

In English we often mean for *or* to be **exclusive**: one or the other, but not both. In math, however, *or* is usually **inclusive**: one or the other, or both. The thing we are including, or excluding is the "both" option.

Example 5: Determine whether each *or* statement is inclusive or exclusive.

a. Would you like a chicken or vegan meal?
b. We want to hire someone who speaks Spanish or Chinese.
c. Are you going to wear sandals or tennis shoes?
d. Are you going to visit Thailand or Vietnam on your trip?

The first *or* statement is a choice of one or the other, but not both, so it is exclusive. The second statement is inclusive because they could find a candidate who speaks both languages. The third statement is exclusive because you can't wear both at the same time. The fourth statement is inclusive because you could visit both countries on your trip.

Conditional Statements (if, then)

A **conditional statement** connects two propositions with *if, then*. An example of a conditional statement would be "*If* it is raining, *then* we'll go to the mall."

The statement "If it is raining," may be true or false for any given day. If the condition is true, then we will follow the course of action and go to the mall. If the condition is false, though, we haven't said anything about what we will or won't do.

Basic Truth Tables

In logic we can use a **truth table** to analyze a complex statement by summarizing all the possibilities and their **truth values** (true or false). To do this, we break the statement down to its smallest elements, the propositions. Then we can see the outcome of the complex statement for all possible combinations of true and false for the propositions.

For example, let's work with two propositions:

R: You paid your rent this month.
E: You paid your electric bill this month.

We will use these two propositions to demonstrate the truth tables for *not*, *and*, and *or*.

To set up a truth table, we list all the possible truth value combinations in a systematic way. The standard way of doing this is to make the first column half true, then half false, then cut the pattern in half with each succeeding column. For two propositions, the first two columns are shown to the right.

Truth Table Setup for Two Propositions

R	*E*	
T	T	
T	F	
F	T	
F	F	

The four possible combinations are

Row 1: You have paid your rent and electric bill
Row 2: You have paid your rent but not your electric bill
Row 3: You have not paid your rent but you have paid your electric bill
Row 4: You haven't paid either your rent or electric bill (yet).

Once we fill in the starting columns, we add additional columns for the more complex statements. We can add as many columns as needed. Below are the basic truth tables for *not*, *and,* and *or*.

Basic Truth Tables

Not

R	*not R*
T	F
F	T

In the *not R* column, the truth value is the opposite of the value for *R*. For example, if *R* is true (you paid your rent) then *not R* (you did not pay your rent) is false.

And

R	*E*	*R and E*
T	T	T
T	F	F
F	T	F
F	F	F

In the *R and E* column, you must have paid both your rent and electric bill. Otherwise R *and E* is false.

Or

R	*E*	*R or E*
T	T	T
T	F	T
F	T	T
F	F	F

In the *R or E* column, you must have paid either your rent or electric bill, or both (inclusive or). Otherwise *R or E* is false.

Conditional Truth Tables

We talked about conditional statements (*if, then* statements), earlier. In logical arguments the first part (the *"if"* part) is usually a **hypothesis** and the second part (the *"then"* part) is a **conclusion**.

To understand the truth table values for a conditional statement it is helpful to look at an example. Let's say a friend tells you, "If you post that photo to Facebook, you'll lose your job." Under what conditions can you say that your friend was wrong?

There are four possible outcomes:

1. You post the photo and lose your job
2. You post the photo and don't lose your job
3. You don't post the photo and lose your job
4. You don't post the photo and don't lose your job

The only case where you can say your friend was wrong is the second case, in which you post the photo but still keep your job.

Your friend didn't say anything about what would happen if you didn't post the photo, so you can't say the last two statements are wrong. Even if you didn't post the photo and lost your job anyway, your friend never said that you were guaranteed to keep your job if you didn't post it.

The four cases above correspond to the four rows of the truth table. For this truth table we will use P for "posting the photo," and L for "losing your job."

Truth table for a conditional statement

P	*L*	If *P*, then *L*
T	T	T
T	F	F
F	T	T
F	F	T

If the hypothesis (the "if" part) is false, we cannot say that the statement is a lie, so the result of the third and fourth rows is true. Notice that we are using a double negation in this explanation.

We are using the words *and*, *or*, *not* and *if then* in this book, but if you look up other resources on truth tables you are likely to see these symbols.

Symbols used in other resources

A and B is written $A \wedge B$	*A or B* is written $A \vee B$
not A is written $\sim A$	*If A, then B is written* $A \rightarrow B$

Truth Tables for Complex Statements

Truth tables really become useful when we analyze more complex statements. In this case we will have several columns. It helps to work from the inside out and create a column in the table for each intermediate statement.

Example 6: Create a truth table for the statement *A or not B*

When we create the truth table, we start with columns for the propositions, *A* and *B*. Then we add a column for *not B* because that is part of the final statement. Our last column is the final statement *A or not B*.

A	*B*	*not B*	*A or not B*
T	T		
T	F		
F	T		
F	F		

To complete the third column, *not B*, we take the opposite of the *B* column. Then to complete the fourth column, we only look at the *A* and the *not B* columns and compare them using *or*.

A	*B*	*not B*	*A or not B*
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

Truth Tables with Three Propositions

To create a truth table with three propositions we need eight rows for all the possible combinations. We will first determine the columns we need to get to our final statement. Then we will fill in the first three columns using the same methodology as before. Start with half true, half false, then cut the pattern in half each time.

Example 7: Create a truth table for the statement *A and not (B or C)*

First let's figure out the columns we will need. We have *A*, *B*, *C*, then we need the statement in the parentheses, *(B or C)*. Then we need the negation of that column, *not (B or C)*. Then we conclude with our final statement, *A and not (B or C)*.

Here is the initial table:

A	*B*	*C*	*B or C*	*not (B or C)*	*A and not (B or C)*
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

Now we complete the columns one at a time. We use the *B* column and *C* column to complete *B or C*. Then *not (B or C)* is the opposite of that column. For the final column we only need to look at the first and fifth columns, shaded in blue, with *and*. Here is the completed table.

A	*B*	*C*	*B or C*	*not (B or C)*	*A and not (B or C)*
T	T	T	T	F	F
T	T	F	T	F	F
T	F	T	T	F	F
T	F	F	F	T	T
F	T	T	T	F	F
F	T	F	T	F	F
F	F	T	T	F	F
F	F	F	F	T	F

For this statement A must be true and neither B or C can be true, so it is only true in the fourth row. For an example of this statement, let's define these propositions in the context of professional baseball:

Let $A =$ Anaheim wins, $B =$ Baltimore wins, $C =$ Cleveland wins.

Suppose that Anaheim will make the playoffs if: (1) Anaheim wins, and (2) neither Boston nor Cleveland wins. TFF is the only scenario in which Anaheim will make the playoffs.

Example 8: Construct a truth table for the statement *if m and not p, then r.*

First, it may help to add parentheses to help you clarify the order. Our statement could also be written, *if (m and not p), then r*. To build this table, we will build the statement in parentheses and then repeat the *r* column after it. It's easier to read the conditional statement from left to right. Here are the columns for the table:

m	*p*	*r*	*not p*	*m and not p*	*r*	*If (m and not p), then r*
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

For the fourth column, we take the opposite of *p*. Then we use the first and fourth columns to complete *m and not p*. With the *r* column repeated we can use columns five and six to complete our conditional statement. Here is the completed table:

m	*p*	*r*	*not p*	*m and not p*	*r*	*If (m and not p), then r*
T	T	T	F	F	T	T
T	T	F	F	F	F	T
T	F	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	T
F	T	F	F	F	F	T
F	F	T	T	F	T	T
F	F	F	T	F	F	T

When m is true, p is false, and r is false—the fourth row of the table—then the hypothesis m and not p will be true, but the conclusion is false, resulting in an invalid conditional statement; every other case gives a true result.

> If you want a real-life situation that could be modeled by if m and not p, then r, consider this:
>
> Let $m =$ we order meatballs, $p =$ we order pasta, and $r =$ Ruba is happy.
>
> The statement if *m and not p*, then r is, "if we order meatballs and don't order pasta, then Ruba is happy". If m is true (we order meatballs), p is false (we don't order pasta), and r is false (Ruba is not happy), then the statement is false, because we satisfied the premise, but Ruba did not satisfy the conclusion.

In this section we have discussed propositions, logical connectors and truth tables. In the next section, we will look at set relationships before we analyze arguments.

Exercises 1.1

1. Which of the following are propositions?
    a. Pigs can fly.
    b. What?
    c. I don't know.
    d. I like tofu.

2. Which of the following are propositions?
    a. How far?
    b. Portland is not in Oregon.
    c. Portland Community College.
    d. It is raining.

3. Write the negation of each proposition.
    a. I ride my bike to campus.
    b. Portland is not in Oregon.

4. Write the negation of each proposition.
    a. You should see this movie.
    b. Lashonda is wearing blue.

5. Write a proposition that contains a double negative.

6. Write a proposition that contains a triple negative.

7. For each situation, decide whether the "or" is most likely exclusive or inclusive.
    a. An entrée at a restaurant includes soup or salad.
    b. You should bring an umbrella or a raincoat with you.

8. For each situation, decide whether the "or" is most likely exclusive or inclusive.
    a. We can keep driving on I-5 or get on I-405 at the next exit.
    b. You should save this document on your computer or a flash drive.

9. Translate each statement from symbolic notation into English sentences. Let *A* represent "Elvis is alive" and let *K* represent "Elvis is the King".
    a. *Not A*
    b. *A or K*
    c. *Not A and K*
    d. *If K, then not A*

10. Translate each statement from symbolic notation into English sentences. Let *A* represent "It rains in Oregon" and let *B* represent "I own an umbrella".
    a. *Not B*
    b. *A and not B*
    c. *If A, then B*
    d. *If not B, then A*

Create a Truth Table for each statement.

11. *A and not B*

12. *Not (not A or B)*

13. *Not (A and B and C)*

14. *Not A or (not B and C)*

15. *Not (A and B) or C*

16. *(A or B) and (A or C)*

17. *If A and B, then C*

18. *If A or B, then not C*

19. *If A and C, then not A*

20. *If B or C, then (A and B)*

Section 1.2 Sets and Venn Diagrams

Sets

It is natural for us to classify items into groups, or **sets**, and consider how they interact with each other. In this section, we will use sets and Venn diagrams to visualize relationships between groups and represent survey data.

A **set** is a collection of items or things. Each item in a set is called a **member** or **element**.

Example 1:

a. The numbers 2 and 42 are elements of the set of all even numbers.

b. MTH 105 is a member of the set of all courses you are taking.

A set consisting entirely of elements of another set is called a **subset.** For instance, the set of numbers 2, 6, and 10 is a subset of the set of all even numbers.

Some sets, like the set of even numbers, can be defined by simply describing their contents. We can also define a set by listing its elements using **set notation.**

Set Notation

Set notation is used to define the contents of a set. Sets are usually named using a capital letter, and its elements are listed *once* inside a set of curly brackets.

For example, to write the set of primary colors using set notation, we could name the set C for colors, and list the names of the primary colors in brackets: $C=$ {red, yellow, blue}. In this case, the set C is a subset of all colors. If we wanted to write the list of our favorite foods using set notation, we could write $F=$ {cheese, raspberries, wine}. And yes, wine is definitely an element of some food group!

Example 2: Julia, Keenan, Jae and Colin took a test. They got the following scores: 70, 95, 85 and 70. Let P be the set of test takers and S be the set of test scores. List the elements of each set using set notation.

In this example, the set of people taking the test is $P=$ {Julia, Keenan, Jae, Colin}, and the set of test scores is $S=\{70, 85, 95\}$. Notice in this example that even though two people scored a 70 on the test, the score of 70 is only listed once.

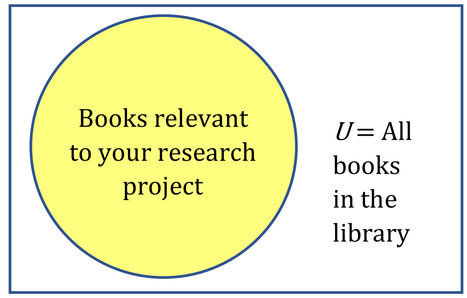
It is important to note that when we write the elements of a set in set notation, there is no order implied. For example, the set $\{1, 2, 3\}$ is equivalent to the set $\{3, 1, 2\}$. It is conventional, however, to list the elements in order if there is one.

The Universal Set

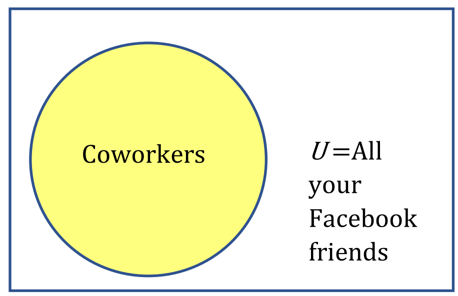
The **universal set** is the set containing every possible element of the described context. Every set is a therefore a subset of the universal set. The universal set is often illustrated by a rectangle labeled with a capital letter U. Subsets of the universal set are usually illustrated with circles for simplicity, but other shapes can be used.

Example 3:

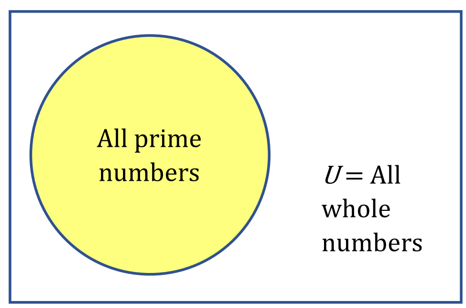
a. If you are searching for books for a research project, the universal set might be all the books in the library, and the books in the library that are relevant to your research project would be a subset of the universal set.



b. If you are wanting to create a group of your Facebook friends that are coworkers, the universal set would be all your Facebook friends and the group of coworkers would be a subset of the universal set.



c. If you are working with sets of numbers, the universal set might be all whole numbers, and all prime numbers would be a subset of the universal set.



The Null Set

It is possible to have a set with nothing in it. This set called the **null set** or **empty set**. It's like going to the grocery store to buy your favorite foods and realizing you left your wallet at home. You walk away with an empty bag. The set of items that you bought at the grocery store would written in set notation as $G = \{ \ \}$, or $G = \emptyset$.

Intersection, Union, and Complement (And, Or, Not)

Suppose you and your roommate decide to have a house party, and you each invite your circle, or set, of friends. When you combine your two sets of friends, you discover that you have some friends in common.

The set of friends that you have in common is called the **intersection.** The **intersection** of two sets contains only the elements that are in both sets. To be in the intersection of set A and B, an element needs to be in both set A **and** set B.

The set of all friends that you and your roommate have invited is called the **union.** The **union** of two sets contains all the elements contained in either set (or both). To be in the union of set A and set B, an element must to be contained in just set A, just set B, **or** in the intersection of sets A and B. Notice that in this case that the *or* is inclusive.

What about the people who were *not* invited to the party and showed up anyway? They are not elements of your set of invited friends. Nor are they an element of your roommate's set of invited friends. These uninvited party crashers are the **complement** to your set of invited friends. The complement of a set A contains everything that is *not* in the set A. To be in the complement of set A, an element **cannot** be in set A, but it will be an element of the universal set.

Example 4: Consider the sets: $A = \{\text{red, green, blue}\}$, $B = \{\text{red, yellow, orange}\}$, and $C = \{\text{red, orange, yellow, green, blue, purple}\}$

a. Determine the set A intersect B, and write it in set notation.

   The intersection contains the elements in both sets: A intersect $B = \{\text{red}\}$

b. Determine the set A union B, and write it in set notation.

   The union contains all the elements in either set: A union $B = \{\text{red, green, blue, yellow, orange}\}$. Notice we only list red once.

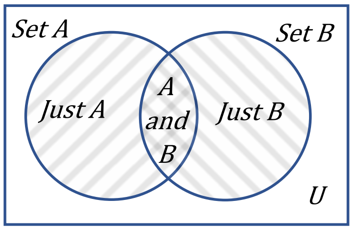
c. Determine the intersection of A complement and C and write it in set notation.

   Here we are looking for all the elements that are *not* in set A and are in set C: A complement intersect $C = \{\text{orange, yellow, purple}\}$

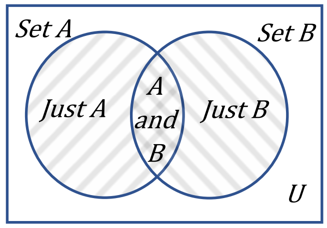
Venn Diagrams

Venn diagrams are used to illustrate the relationships between two or more sets. To create a Venn diagram, start by drawing a rectangle to represent the universal set. Next draw and label overlapping circles to represent each of your sets. Most often there will be two or three sets illustrated in a Venn diagram. Finally, if you are given elements, fill in each region with its corresponding elements.

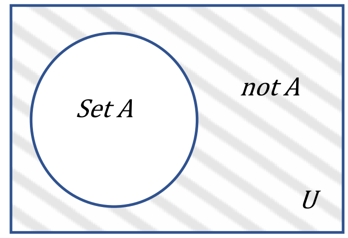
Venn diagrams are also a great way to illustrate intersections, unions and complements of sets as shown below.



The **intersection** is where the shading of the two sets overlaps in the center. It contains the elements of A **and** B.



The **union** includes all elements of A **or** B or both. It contains all three of the shaded regions.



The **complement** of set A includes all the elements **not** in A. It is the shaded region outside the set of A, but within the universal set.

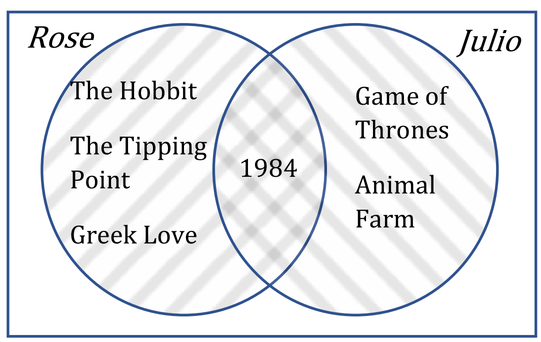
Here is an example of how to draw a Venn Diagram.

Example 5: Let J be the set of books Julio read this summer and let R be the set of books Rose read this summer. Draw a Venn diagram to show the sets of books they read if Julio read Game of Thrones, Animal Farm and 1984, and Rose read The Hobbit, 1984, The Tipping Point, and Geek Love.

To create a Venn diagram showing the relationship between the set of books Julio read and the set of books Rose read, first draw a rectangle to illustrate the universal set of all books.

Next draw two overlapping circles, one for the set of books Julio read and one for the set of books Rose read. Since both Rose and Julio read 1984, we place it in the overlapping region (the intersection).

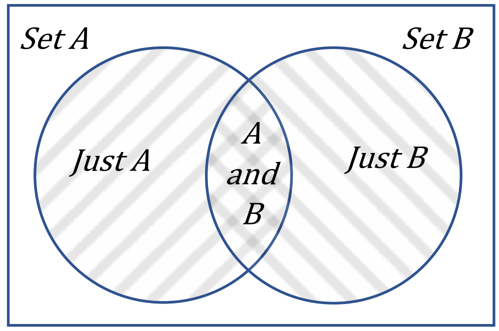
All the books that Rose read will lie in her circle, in one of the two regions that make up her set. Likewise for the books Julio read. Since we have already filled in the overlapping region, we put the books that only Rose read in her circle's "cresent moon" section, and we put the books that only Julio read in his circle's "cresent moon" section. The resulting diagram is shown below.



Example 6: In the last section we discussed the difference between **inclusive "or"** and **exclusive "or**." In common language, "or" is usually exclusive, meaning the set A or B includes just A or just B but not both. In logic, however, "or" is

inclusive, so the set A or B includes just A, just B, or both. The difference between the inclusive and exclusive "or" can be illustrated in a Venn, as shown below.

Inclusive OR includes all three shaded regions, Just A, Just B, and A and B.



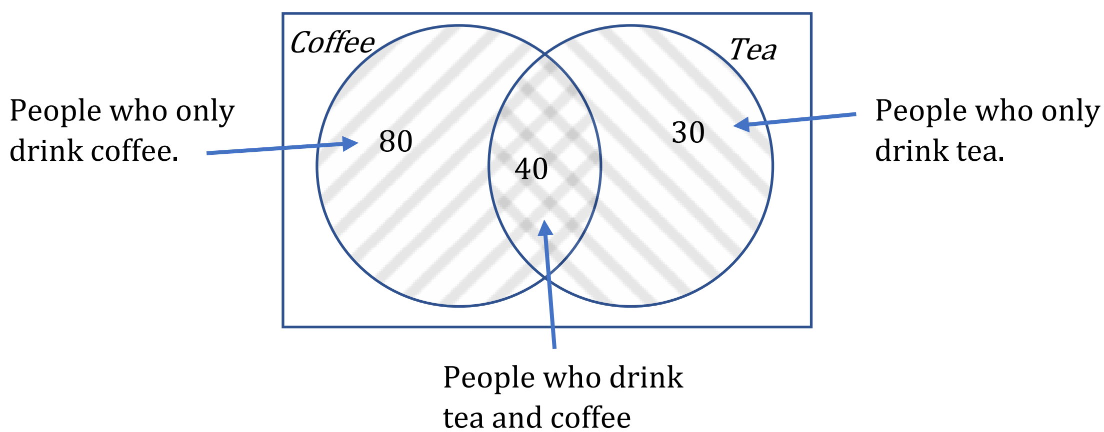
Exclusive OR includes the single shaded regions, Just A and Just B, but not the intersection. You can have A or B but not both.

Illustrating Data

We can also use Venn diagrams to illustrate quantities, data, or frequencies.

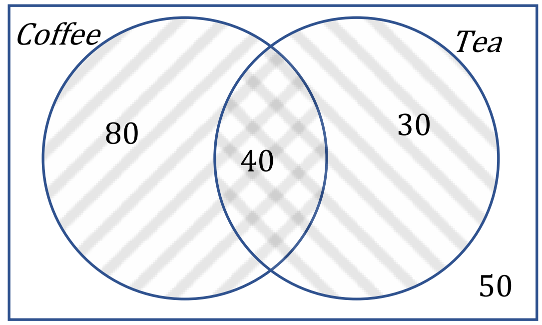
Example 7: A survey asks 200 people, "What beverage(s) do you drink in the morning?" and offers three choices: tea only, coffee only, and both coffee and tea. Thirty report drinking only tea in the morning, 80 report drinking only coffee in the morning, and 40 report drinking both. How many people drink tea in the morning? How many people drink neither tea nor coffee?

To answer this question, let's first create a Venn diagram representing the survey results. Placing the given values, we have the following:



The universal set should include all 200 people surveyed, but we only have 150 placed so far. The difference between what we have placed so far, and the 200 total is the number of people who drink neither coffee nor tea. These 200 – 150 = 50 people are placed outside of the circles but within the rectangle since they are still included in the universal set.

The number of people who drink tea in the morning includes everyone in the tea circle. This includes those who only drink tea and those who drink both tea and coffee. Thus, the number of people who drink tea is 40 + 30 = 70.



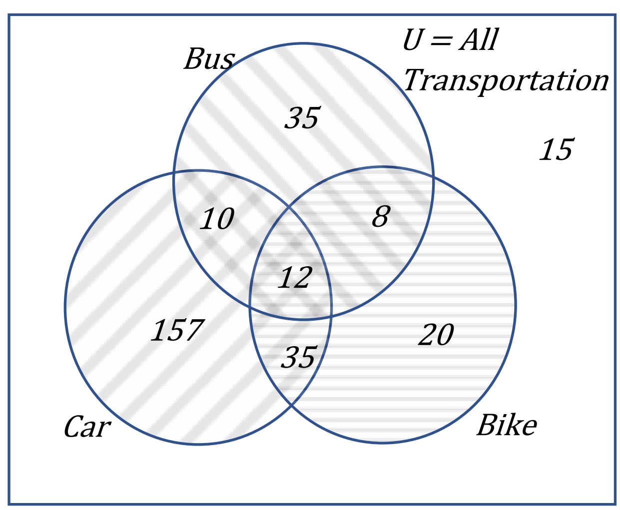
Here is an example of a Venn diagram with three sets.

Example 8: In a survey, adults were asked how they travel to work. Below is the recorded data on how many people took the bus, biked, and/or drove to work. Draw and label a Venn diagram using the information in the table.

Travel Options	Frequency
Just Car	157
Just Bike	20
Just Bus	35
Car and Bike only	35
Car and Bus only	10
Bus and Bike only	8
Car, Bus and Bike	12
Neither Car, Bus nor Bike	15
Total	292

To fill in the Venn diagram, we will place the 157 people who only drive a car in the car set where it does not overlap with any other modes of transportation. We can fill in the numbers 20 and 35 in a similar way.

Then we have the overlap of two modes of transportation only. There are 35 people who use their car and bike only, so they go in the overlap of those two sets, but they do not take the bus, so they are outside of the bus set. Similarly, we can enter the 10 and 8. There are 12 people who use all three modes, so they are in the intersection of all three sets. There are 15 people who do not use any of the three modes, so they are placed outside the circles but inside the universal set of all modes of transportation. Here is the completed Venn diagram.



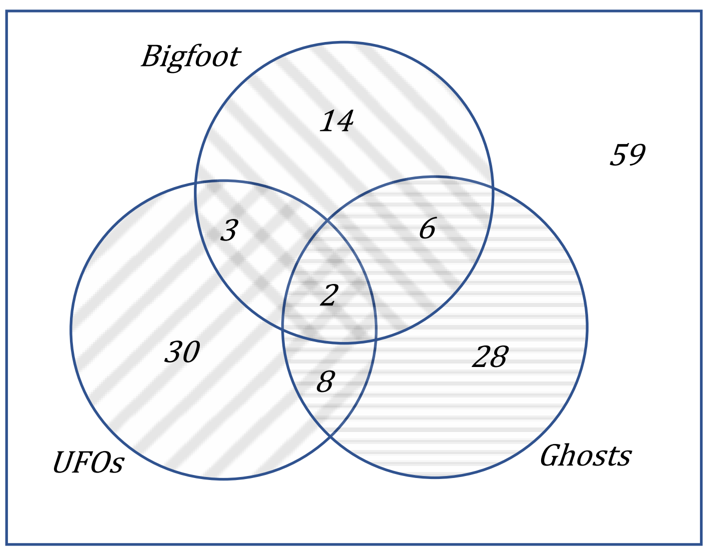
Example 9: One hundred fifty people were surveyed and asked if they believed in UFOs, ghosts, and Bigfoot. The following results were recorded.

- 43 believed in UFOs
- 44 believed in ghosts
- 25 believed in Bigfoot
- 10 believed in UFOs and ghosts
- 8 believed in ghosts and Bigfoot
- 5 believed in UFOs and Bigfoot
- 2 believed in all three

Draw and label a Venn diagram to determine how many people believed in at least two of these things.

Starting with the intersection of all three circles, we work our way out. The number in the center is 2, since two people believe in UFO's, ghosts and Bigfoot. Since 10 people believe in UFOs and Ghosts, and that includes the 2 that believe in all three, that leaves 8 that believe in only UFOs and Ghosts.

We work our way out, filling in all the regions. Once we have, we can add up all those regions, getting 91 people in the union of all three sets. This leaves 150 – 91 = 59 who believe in none.



Then to answer the question of how many people believed in **at least two** (two or more), we add up the numbers in the intersections, $8 + 2 + 3 + 6 = 19$ people.

Qualified Propositions

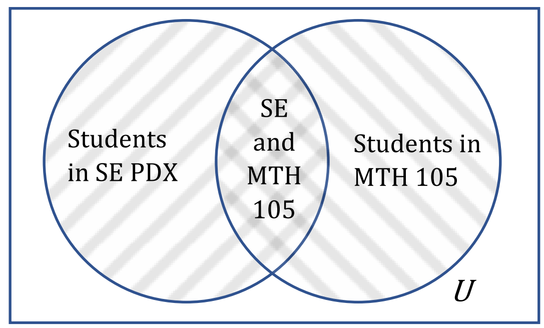
A **qualified proposition** is a statement that asserts a relationship between two sets. The three relationships we will be looking at in this section are "some" (some elements are shared between the two sets), "none" (none of the elements are shared between the two sets), and "all" (all elements of one set are contained in the other set). These relationships are especially important in evaluating arguments.

Overlapping Sets

Sets overlap if they have members in common. The Venn diagram examples we have looked at in this section are **overlapping** sets.

Example 10: The set of students living in SE Portland and the set of students taking MTH 105.

Qualified Proposition: "**Some** students who live in SE Portland take MTH 105."

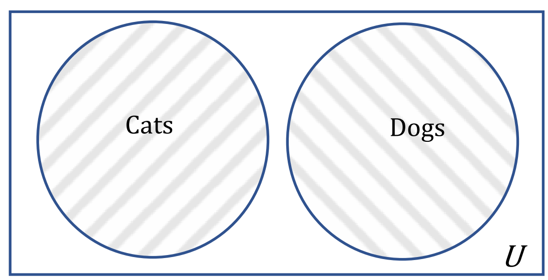


Disjoint Sets

Sets are **disjoint** if they have no members in common.

Example 11: The set of Cats and the set of Dogs.

Qualified Proposition: "**No** cats are Dogs."

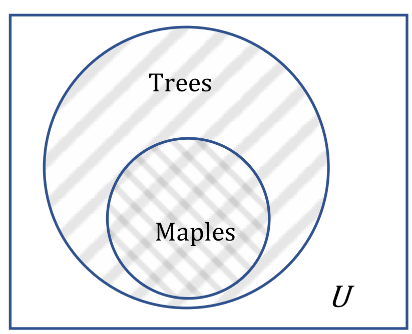


Subsets

If a set is completely contained in another set, it is called a **subset**.

Example 12: The set of all Trees and the set of Maples Trees.

Qualified Proposition: "**All** Maples are Trees."



Exercises 1.2

1. List the elements of the set "The letters of the word Mississippi."
2. List the elements of the set "Months of the year."
3. Write a verbal description of the set {3, 6, 9}.
4. Write a verbal description of the set {a, i, e, o, u}.
5. Is {1, 3, 5} a subset of the set of odd numbers?
6. Is {A, B, C} a subset of the set of letters of the alphabet?

Create a Venn diagram to illustrate each of the following:

7. A survey was given asking whether people watch movies at home from Netflix, Redbox, or a video store. Use the results to determine how many people use Redbox.
   - 52 only use Netflix, 62 only use Redbox
   - 24 only use a video store, 16 use only a video store and Redbox
   - 48 use only Netflix and Redbox, 30 use only a video store and Netflix
   - 10 use all three, 25 use none of these

8. A survey asked buyers whether color, size, or brand influenced their choice of cell phone. The results are below. How many people were influenced by brand?
   - 5 said only color, 8 said only size
   - 16 said only brand, 20 said only color and size
   - 42 said only color and brand, 53 said only size and brand
   - 102 said all three, 20 said none of these

9. Use the given information to complete a Venn diagram, then determine: a) how many students have seen exactly one of these movies, and b) how many have seen only *Star Wars*.
   - 18 have seen *The Matrix* (*M*), 24 have seen *Star Wars (SW)*
   - 20 have seen *Lord of the Rings (LotR),* 10 have seen *M* and *SW*
   - 14 have seen *LotR* and *SW,* 12 have seen *M* and *LotR*
   - 6 have seen all three

10. A survey asked people what alternative transportation modes they use. Use the data to complete a Venn diagram, then determine: a) what percentage of people only ride the bus, and b) what percentage don't use any alternate transportation.
    - 30% use the bus, 20% ride a bicycle
    - 25% walk, 5% use the bus and ride a bicycle
    - 10% ride a bicycle and walk, 12% use the bus and walk
    - 2% use all three

Given the qualified propositions: A) Determine the two sets being described B) Determine if the sets described are Subsets, Overlapping Sets or Disjoint sets. C) illustrate the situation using sets.

11. All Terriers are dogs.

12. Some Mammals Swim. (The second set is not clearly defined but is implied)

13. No pigs can fly.

14. All children are young.

15. Some friends remember your birthday.

16. No lies are truths.

Section 1.3 Describing and Critiquing Arguments

Logical Arguments

A **logical argument** is a claim that a set of **premises** support a **conclusion.** It is possible for a logical argument to have one or many premises, but there must be one conclusion. In this section we will look at types of arguments and how to determine the strength, validity and/or soundness of each type.

There are two types of arguments we will explore in this section: **inductive** and **deductive** arguments.

Inductive and Deductive Arguments

To better understand the difference between inductive and deductive arguments, let's start by looking at a couple of examples.

Example 1: Consider the following argument:

When I went to the store last week, I forgot my wallet, and I forgot it again when went back today. I always forget my wallet when I go to the store.

Before we analyze an argument, it is helpful to precisely state its premises and its conclusion. Most arguments you encounter in the real world won't be stated in a precise "premise, premise, conclusion" form. Sometimes the conclusion will be stated before the premises, or the premises will be hidden within a bunch of rhetoric.

To begin our analysis of this first argument, let's first rewrite it in a more precise "premise, premise, conclusion" form.

Premise: I forgot my wallet when I went to the store last week.

Premise: I forgot my wallet when I went to the store today.

Conclusion: I always forget my wallet when I go to the store.

Notice that both premises make a claim about a **specific** instance – the specific instance last week when I forgot my wallet, and the specific instance today when I forgot my wallet. The conclusion, on the other hand, states what we can expect to happen more generally.

Now let's consider a different argument:

Example 2: Henry must know CPR because he is a nurse and all nurses know CPR.

Just as we did for the last example, let's rewrite the argument in its "premise, premise, conclusion" form:

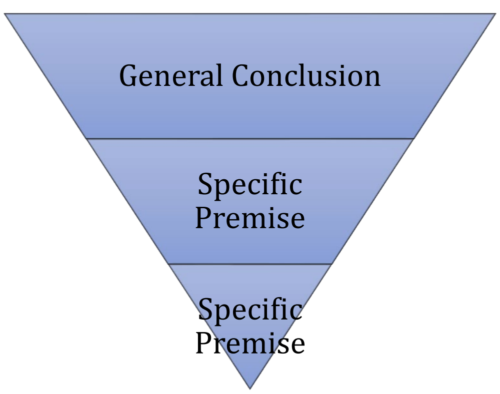
Premise: All nurses know CPR.

Premise: Henry is a nurse.

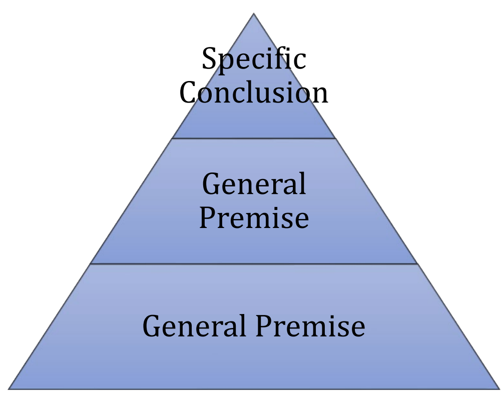
Conclusion: Henry knows CPR.

Unlike the first argument where the premises were specific and the conclusion was general, this argument's first premise is a general statement and the conclusion is specific. We can determine whether an argument is inductive or deductive by looking at which part of the argument is general and which is specific. In the first example, the premises were specific and the conclusion was more general. This is an example of an **inductive** argument. In the second example, it was the premises that were more general and the conclusion that was specific. This is an example of a **deductive** argument.

In general, an **inductive argument** uses a collection of *specific* examples (i.e. data) as its premises and uses them to propose a *general* conclusion, while a **deductive argument** uses a collection of *general* statements (i.e. definitions) as its premises and uses them to propose a *specific* conclusion. You can see the difference in the pyramids below. We start with the premises at the bottom and build up to the conclusion.



Inductive Argument
(Specific to General)



Deductive Argument
(General to Specific)

Example 3:
Rewrite the following arguments in a precise "premise, premise, conclusion" form, and determine if the argument is inductive or deductive.

a. A number is prime if it is only divisible by itself and one. Since the number 13 is only divisible by itself and one, 13 must be prime.

Premise: If a number is only divisible by itself and one, the number is prime.

Premise: The number 13 is only divisible by itself and one.

Conclusion: 13 is prime.

Since the premises are general definitions and properties of numbers and the conclusion is a specific statement about the number 13, the argument is **deductive**.

b. Juan's dog Goober is having puppies. All three of Goober's previous litters have had 5 puppies so Goober is bound to have 5 puppies in this litter as well.

Premise: Goober is having puppies.

Premise: Goober's last three litters had 5 puppies.

Conclusion: Goober's current litter will have 5 puppies.

This is an example of an **inductive argument** since it uses specific experiences/instances as its premises, and its conclusion is a general expectation based on those specific experiences.

Evaluating Arguments

Inductive arguments cannot be proven. The best we can do is evaluate the **strength** of the argument based on the evidence it provides.

A strong inductive argument is one that is well supported by its premises, while a weak inductive argument is one whose premises do a poor job of supporting the conclusion. The strength of an inductive argument is subjective, because where one person sees a strong argument, another may see a weak argument. Additionally, the strength and truth of an argument are not necessarily related; it is possible to have a weak argument that is true, and a strong argument that is false.

<u>Example 4</u>:
Determine the strength of the inductive argument.

James Franco, Jodie Foster, Jennifer Lawrence, and Jack Nicholson have all won Academy Awards for acting. Actors whose names start with J are bound to win an Academy Award.

The inductive argument provides a number of specific cases as evidence for the conclusion. However, we would not be surprised if a J-named actor *did not* win an Academy Award, so the argument is weak

Deductive arguments, on the other hand, can be proven and their validity and soundness can be evaluated. The **validity of the argument** is based on whether the conclusion follows logically from the premises, while the **soundness of the argument** is based on whether or not the premises are true. An argument cannot be sound if it is not valid, even if the premises seem reasonable.

Evaluating Deductive Arguments Using Sets

One way to determine whether a deductive argument is valid is to illustrate the premises of the argument using sets and see if the conclusion logically follows if we assume the premises to be true.

Example 5:
Use a set diagram to determine whether the argument is valid. If the argument is valid, determine if it is also sound.

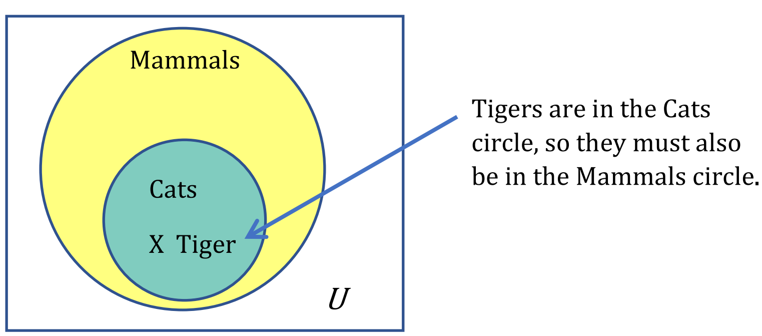
a. "All cats are mammals and a tiger is a cat, so a tiger is a mammal."
First let's write the argument in its "premise, premise, conclusion" form. For the problems we will be looking at, you will want to write the first premise as a **qualified proposition** (some, none, all) since this will form the basic structure of our diagram.

Premise: All cats are mammals.

Premise: A tiger is a cat.

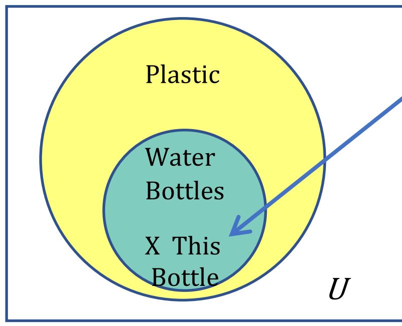
Conclusion: A tiger is a mammal.

From the first premise we know that all cats lie inside the set of mammals (cats are a subset of mammals). From the second premise, we know that tigers lie inside the set of cats (marked with an X), and therefore also lie within the set of mammals.



This argument is valid because we were able to show that the conclusion follows logically from the premises. The argument is also sound since the premises "all cats are mammals" and "a tiger is a cat" are true.

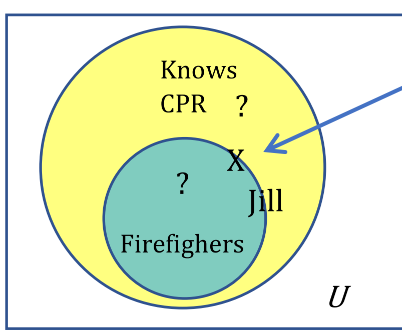
b. "All water bottles are plastic. This is a water bottle, so it must be plastic." From the first premise we know that all water bottles lie inside the set of plastic items (water bottles are a subset of plastic). From the second premise, we know that this particular water bottle must lie within the plastic items set.



Any water bottle will be inside the plastic circle also.

This argument is valid because we were able to show that the conclusion follows logically from the premises. But the argument is not sound because the premise that all water bottles are plastic is not true. There are many versions of glass and metal bottles that are evidence that the first premise is not true. This argument is valid but not sound.

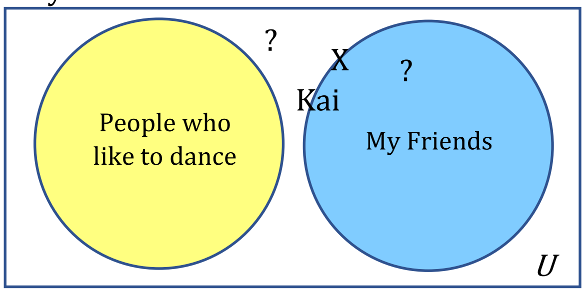
c. "All firefighters know CPR. Jill knows CPR, so Jill must be a firefighter." From the first premise we know that all firefighters lie inside the set of those who know CPR (firefighters are a subset of people who know CPR). From the second premise, we know that Jill is a member of the set of those who know CPR, but we do not have enough information to know whether she is also a member of set of firefighters.



Jill is somewhere in the "Knows CPR" circle, but it is not clear if she is in the firefighter circle or not. We will put the X for Jill on the border of the two regions.

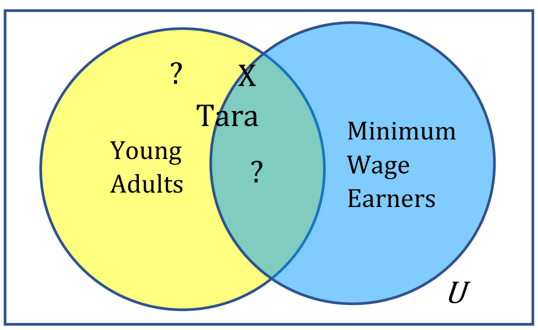
Since we cannot determine which group Jill must be a part of, the argument is invalid. The statement that Jill is a firefighter does not follow logically from the premises that "all firefighters know CPR" and that "Jill knows CPR". Since the argument is not valid, it cannot be sound.

d. "None of my friends like dancing. Kai doesn't like dancing. Therefore, Kai is my friend."



Because it said "none" we draw disjoint sets – one set for my friends and a second set for people who like to dance. The second premise tells us that Kai doesn't like to dance so they're not in the set of people who like to dance. However, we can't put Kai in the set of my friends either. They could be my friend, or someone I don't know who happens to not like dancing. Therefore, the conclusion is **not valid**. And therefore, is also **not sound**.

e. "Some young adults make minimum wage and Tara is a young adult. Therefore, Tara makes minimum wage."



Because it said "some" we draw overlapping sets. The second premise tells us to put Tara in the set of young adults, but it doesn't tell us if she makes minimum wage or not. So, like the previous example we cannot determine which region she is in. She could make minimum wage, or she could also make more. Therefore, the conclusion is **not valid** and therefore, **not sound.**

Exercises 1.3

Rewrite each of the following arguments in their "premise, premise, conclusion" form, and determine whether the argument is inductive or deductive. If the argument is inductive, determine its strength. If the argument is deductive, use sets to illustrate and determine the validity of the argument, and state whether the argument is sound.

1. Since all cats are scared of vacuum cleaners and Max is a cat, Max must be scared of vacuum cleaners.

2. Every day for the last year, a plane flew over my house at 2 pm. Therefore, a plane will always fly over my house at 2pm.

3. Kiran collected data on the salaries of their friends. They found that female and nonbinary friends made less than male friends, so they concluded that women and nonbinary people make less than men.

4. Some of these kids are rude. Jimmy is one of these kids. Therefore, Jimmy is rude!

5. All bicycles have two wheels. My friend's Harley-Davidson has two wheels, so it must be a bicycle.

6. Since all chocolate contains nuts and this bar is made of chocolate, it must contain nuts.

7. All students drink a lot of caffeine. Brayer drinks a lot of caffeine, so he must be a student.

8. Over the course of a year, data was collected on the number of students visiting the Cafeteria. On average, there were 15-35 students present in the cafeteria during the peak hours of the data. We can expect there to be between 15 and 35 students in the cafeteria if we go during the peak hours of the day.

9. If a person is on this reality show, they must be self-absorbed. Laura is not self-absorbed. Therefore, Laura cannot be on this reality show.

For each, draw the appropriate illustration of sets (Subset, Disjoint or Overlapping). Then put an X to represent the subject of the conclusion. Or two question marks to illustrate the subject could into two locations. Finally, state if the conclusion is valid.

10. Premise: No apples are pears.
    Premise: A Pink Lady is an apple.
    Conclusion: Therefore, a Pink Lady is not a pear.

11. Premise: All children are young.
    Premise: Tamika is young.
    Conclusion: Therefore, Tamika is a child.

12. Premise: Some goats faint.
    Premise: Fizzy faints.
    Conclusion: Therefore, Fizzy is a goat.

13. Premise: All students who miss more than 25% of class time fail.
    Premise: Claudia failed my class.
    Conclusion: Claudia missed more than 25% of class time.

14. Premise: All students who miss more than 25% of class time fail.
    Premise: Ethan missed more than 25% of class time.
    Conclusion: Ethan failed.

Section 1.4 Logical Fallacies

Logical Fallacies

In the last section we saw that logical arguments are invalid when the premises are not sufficient to guarantee the conclusion, and that even if an argument is valid it may be unsound if the premises are not true. There are other ways that a logical argument my by invalid or unsound. One of the more common ways this can occur is if the argument is a **fallacy**.

A **fallacy** is a type of argument that appears valid but uses a logical error to persuade or deceive. Fallacious arguments are especially common in advertising and politics, so it is important as informed citizens to recognize when we are being presented with a fallacious argument and to not be persuaded by it.

Common Logical Fallacies

There are many logical fallacies, and some go by more than one name. Below we introduce a few of the more common fallacies that you will be asked to recognize by name, but there are many others.

Personal Attack (Ad hominem)

A **personal attack** argument attacks the person making the argument while ignoring the argument itself. A personal attack is not the same as an insult. Rather, a personal attack claims that there is something wrong with the person or group in order to cast doubt on their character and discredit their argument.

> <u>Example 1</u>: "Jane says that whales aren't fish, but she's only in the second grade so she can't be right."
>
> Here the argument is attacking Jane, not the validity of her claim, so this is a personal attack.
>
> <u>Example 2</u>: "Jane says that whales aren't fish, but everyone knows that they're really mammals. She's so stupid."
>
> This certainly isn't very nice, but it is *not* a personal attack since a valid counterargument is made ("they really are mammals") along with a personal insult.
>
> <u>Example 3</u>: "Mr. Smith is a college dropout, so his stance on education reform cannot be trusted."
>
> Here the argument uses the fact that Mr. Smith did not complete their college degree to discredit their ideas on education reform, so it is a personal attack.

Appeal to Ignorance

An **appeal to ignorance** argument assumes something is true because it hasn't been proven false.

> Example 4: "Nobody has proven that photo isn't of Bigfoot, so it must be Bigfoot."
>
> This is an example of an appeal to ignorance since the fact that no one has been able to prove the picture of Bigfoot is false is being used as evidence that it is Bigfoot.

Appeal to Authority

An **appeal to authority** argument attempts to use the authority of a person to prove a claim. An authority could be an expert such as a doctor or scholar, or someone who is admired like a celebrity or sports figure. While an authority can provide strength to an argument, problems can occur when the person's opinion is not shared by other experts, or when the authority is irrelevant to the claim.

> Example 5: "A diet high in bacon can be healthy; Doctor Atkins said so."
>
> Here, an appeal to a doctor's authority is used for the argument. This generally would provide strength to the argument, except that the opinion that eating a diet high in saturated fat runs counter to general medical opinion. More supporting evidence would be needed to justify this claim.
>
> Example 6: "Jennifer Hudson and Oprah lost weight with Weight Watchers, so their program must work."
>
> In this example there is an appeal to the authority of celebrities. While their experience does provide evidence, it provides no more than any other person's experience would.

False Dilemma

A **false dilemma** argument falsely frames an argument as an "either or" choice without allowing for additional options.

> **Example:** "Either those lights in the sky were an airplane or aliens. There are no airplanes scheduled for tonight, so it must be aliens."
>
> This argument is a false dilemma since it ignores the possibility that the lights could be something other than an airplane or aliens.

Straw Man (or Straw Person)

A **straw person** argument involves misrepresenting the argument in an oversimplified, distorted and less favorable way to make it easier to attack.

> Example 7: "Senator Khouri has proposed reducing military spending by 10%. Apparently, she wants to leave us defenseless against attacks by terrorists."

Here the arguer has represented a 10% funding cut as equivalent to leaving us defenseless, making it easier to attack Senator Khouri's position.

Post Hoc

A **post hoc** argument claims that because two things happened sequentially, then the first must have *caused* the second.

Example 8: "Every morning the rooster crows just before dawn. It must be his crow that makes the sun rise."

Here the arguer is saying the rooster caused the sun to rise, but it is more likely that the sun rising caused the rooster to crow.

Example 9: "Today I wore a red shirt and my football team won! I need to wear a red shirt every time they play to make sure they keep winning."

This person is saying their team won because they wore a red shirt. This type of superstition is quite common in sports even though we really know they are unrelated.

Sometimes there may be more than one fallacy that seems reasonable. Consider this argument: "Emma Watson says she's a feminist, but she posed for these racy pictures. I'm a feminist and no self-respecting feminist would do that." Could this be ad hominem, saying that Emma Watson has no self-respect? Could it be appealing to authority because the person making the argument claims to be a feminist? Could it be a false dilemma because the argument assumes that a woman is either a feminist or not, with no gray area in between?

We have described just six of the many types of logical fallacies. Once you learn to recognize these you will also likely become aware of many others. There are many lists of logical fallacies online.

Exercises 1.4

Determine which type of fallacy each argument represents.

1. John Bardeen's work at the Advanced Institute for Physics has progressed so slowly that even his colleagues call him a plodder. Hence, it is prudent at present not to take seriously his current theory relating how strings constitute the smallest of subatomic particles.

2. You will tell the general manager that I made the right choice in dealing with that customer. After all, I'm the shift manager, so my decisions are always right.

3. It was his fault, Officer. You can tell by the kind of car I'm driving and by my clothes that I am a good citizen and would not lie. Look at the rattletrap he is driving and look at how he is dressed. You can't believe anything a dirty, longhaired hippie like that might tell you. Search his car; he probably has pot in it.

4. We can go to the amusement park or the library. The amusement park is too expensive, so we must go to the library.

5. During the Gulf war many American businesses made immense profits. That is an indisputable fact. Therefore, there can be no doubt that American business interests instigated the war.

6. The oven was working fine until you started using it, so you must have broken it.

7. Old man Brown claims that he saw a flying saucer in his farm, but he never got beyond the fourth grade in school and can hardly read or write. He is completely ignorant of what scientists have written on the subject, so his report cannot possibly be true.

8. There are a number of fallacies that were not discussed in this section. Do an internet search for the following fallacies. Provide both a definition and at least one example.

    a. Slippery Slope
    b. Circular Reasoning
    c. Appeal to Emotion
    d. Red Herring
    e. Whataboutism