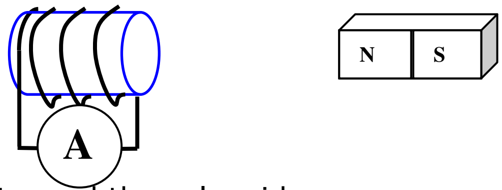
Faraday's Law of Electromagnetic Induction and Lenz's Law

1. For the following scenarios, determine whether the magnetic flux changes or stays the same. If the flux changes: indicate whether it is increasing or decreasing (and in which direction). Explain your answer.

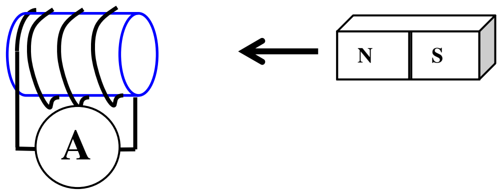
   a. The magnet is held stationary to the solenoid.

Ans. $\frac{d\Phi_B}{dt} = 0$



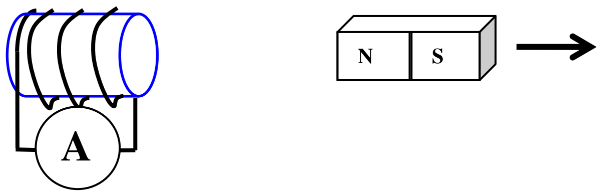
   b. The magnet is moving toward the solenoid.

Ans. $\frac{d\Phi_B}{dt}$ increasing



   c. The magnet is moving away from the solenoid.

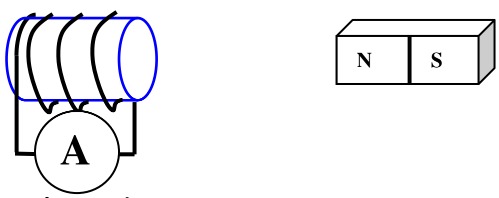
Ans. $\frac{d\Phi_B}{dt}$ decreasing



2. Find the direction of the induced current for the solenoid in the figure below, when the magnet is ______.

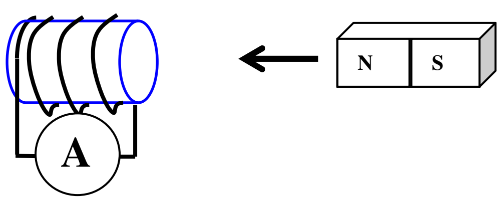
   a. stationary to the solenoid.

Ans. i=0



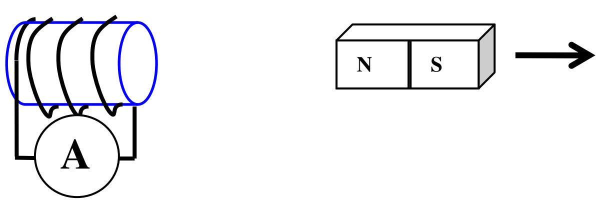
   b. moving toward the solenoid.

Ans. counter-clockwise

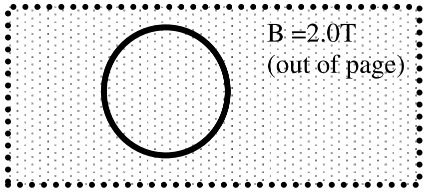


   c. moving away from the solenoid.

Ans. clockwise



3. A circular loop (radius of 10 cm or 0.10 m) is placed in a uniform magnetic field of magnitude, B = 2.0 T, where the face of the loop is perpendicular to the direction of the magnetic field.



a. Determine the magnetic flux through the loop.

Ans. $\Phi_B = \int \vec{B}\cdot d\vec{A} = BA\cos\phi = 0.628\ T\cdot m^2$

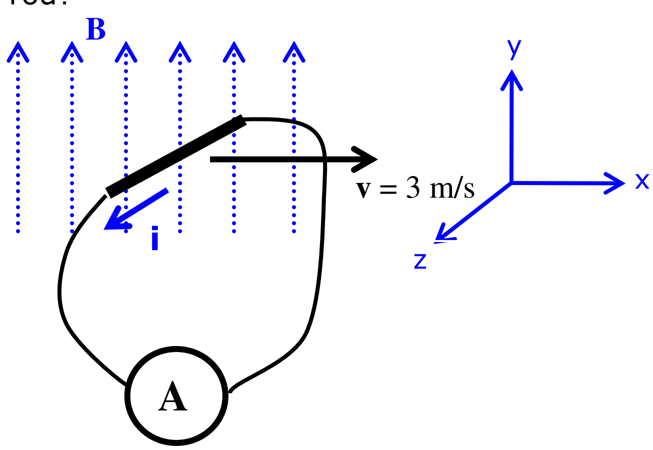
b. The loop is then rotated 90° in 3.0 seconds. What is the magnetic flux through the loop at the end of the 3.0 seconds?

Ans. $\Phi_B = \int \vec{B}\cdot d\vec{A} = BA\cos\phi = 0\ T\cdot m^2$

c. What is the induced emf in the loop during the rotation?

Ans. $\varepsilon = -\frac{d\Phi_B}{dt} = \frac{0.628\ T\cdot m^2}{3\ s} = 0.0209\ \frac{T\cdot m^2}{s}$ (or V)

4. A person moves a 2-m rod at a constant velocity of 3 m/s in a magnetic field, B= 2.0 T. The rod is perpendicular to the direction of the **B** field.



a. What is the direction of induced current in the rod?

Ans. in the +z direction

b. Determine the induced emf in the rod.

Ans. $\varepsilon = \frac{W}{q} = \frac{\vec{F}_B\cdot\vec{L}}{q} = \frac{qvBL}{q} = vBL = 12\ V$

c. The resistance in the rod (and connecting wires) is 2-Ω. What is the current in the rod?

Ans. $i = \frac{\varepsilon}{R} = \frac{12V}{2\Omega} = 6A$

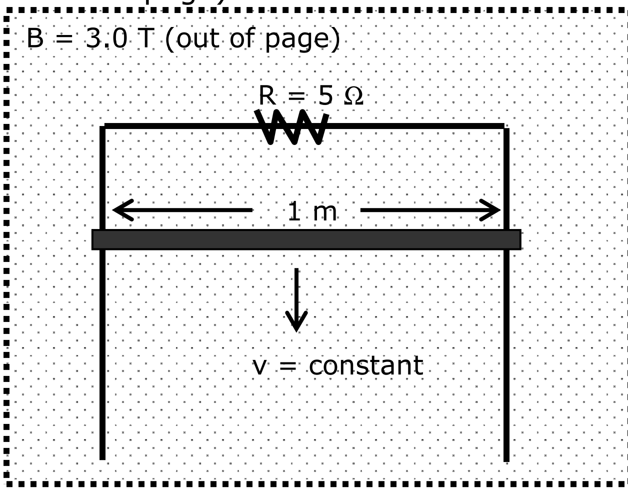
d. Determine the magnitude and direction of the magnetic force acting on the rod.

Ans. $\vec{F}_B = i\vec{L}\times\vec{B} = -\ 24N\ \hat{i}$

e. Determine the force the person exerts on the rod to keep it in motion.

Ans. $\vec{F}_{net} = \vec{F}_B + \vec{F}_{hand} = 0 \Rightarrow \vec{F}_{hand} = -\vec{F}_B = 24N\ \hat{i}$

5. Consider a 1-m conducting rod attached at each end by conducting rails. The rails are connected at the top and the total loop has a resistance of 5-Ω. (see figure below). The rod falls to the ground at a constant velocity, v. The apparatus is inside a constant magnetic field, B = 3.0 T (directed out of the page). The mass of the rod is 0.5kg.



a) What is the magnetic force on the falling rod, due to the magnetic field?

Ans. $\vec{F}_B = \vec{F}_{net} - m\vec{g} = mg\,\hat{j} = 4.9N\,\hat{j}$

b) What is the induced current in the rod?

Ans. $F_B = iLB\sin\phi = 4.9N \Rightarrow i = \dfrac{F_B}{LB} = \dfrac{4.9N}{(1m)(3T)} = 1.63A$

c) What is the induced electromotive force, ε?

Ans. $\varepsilon = iR = (1.63A)(5\Omega) = 8.15\text{ V}$

d) What is the equation for the rate of change of magnetic flux for this problem?

Ans. $\dfrac{d\Phi_B}{dt} = B\dfrac{dA}{dt} = B\dfrac{d(Lh)}{dt} = BL\dfrac{dh}{dt} = BLv = -\varepsilon$

e) How fast is the rod falling?

Ans. $\vec{v} = -\dfrac{\varepsilon}{BL}\hat{j} = -\dfrac{8.15\text{ V}}{3\text{ T}\cdot\text{m}}\hat{j} = -2.72\tfrac{m}{s}\,\hat{j}$

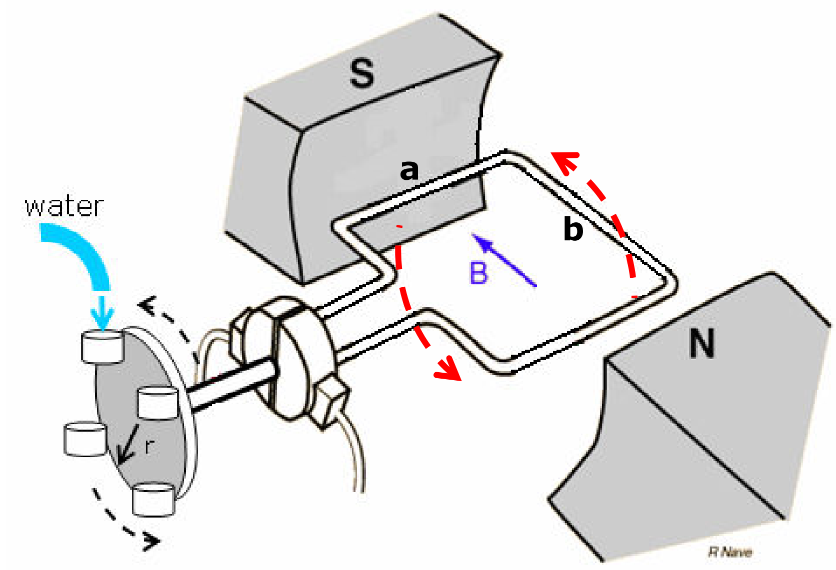
f) When the rail falls for 1 sec, verify that energy is conserved.

Ans. $P_{mg} = P_\varepsilon \rightarrow mgv = \varepsilon i \rightarrow |\vec{v}| = \dfrac{\varepsilon i}{mg} = 2.71\tfrac{m}{s}$, checks with (e)

Generator

6. A water powered generator, shown below, to convert mechanical energy into electrical energy. A rotating wheel receives falling water forcing a wire loop (N=500), located within a constant magnetic field B=0.01 T (as shown), to rotate counter-clockwise at a rate of 150 rpm. The length of the segment normal to the B field (side a) are 0.20 m and the length of the segment parallel to the field (side b) is 0.15 m.

   a. What is the area of the region of the coil within the magnetic field?



Ans.

$$A = ab$$
$$A = (0.15\text{m})(.20\text{m})$$
$$A = 0.030\ \text{m}^2$$

   b. Determine the general equation for the magnetic flux through the coil in terms of area A, B, and angular velocity ω.

Ans. $\Phi_B = \int \vec{B}\cdot d\vec{A} = B\cdot A\cdot \cos\omega t$

   c. What is the angular velocity ω of the rotating coil?

Ans. $\omega = \left(150\,\frac{\text{rot}}{\text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rot}}\right) = 15.7\,\frac{\text{rad}}{\text{s}}$

   d. Calculate the induced electromotive force around the loop.

Ans.

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -N\frac{d}{dt}(B\cdot A\cdot \cos\omega t)$$

$$\varepsilon_{avg} = \frac{NBA\omega\int_0^{\frac{T}{2}}\sin\omega t\ dt}{\frac{T}{2}} = -\frac{2NBA}{T}\cos\omega t\Big|_0^{\frac{T}{2}} = -\frac{4NBA}{T}$$

$$\varepsilon_{avg} = -1.5\ \text{V}$$

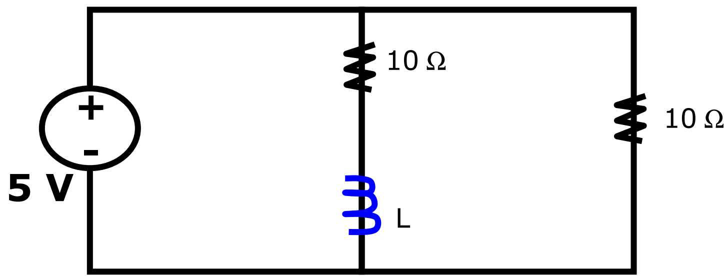
The instantaneous emf is: $\varepsilon(t) = NBA\omega\sin(\omega t + \phi)$

   e. What direction does the induced current flow around the coil? Explain.

Ans. The current will flow clockwise (looking down on the armature), in accordance with RHR.

Self Inductance:

7. A solenoid, r=0.01 m, l=0.03 m (length) and N=100, is in series with a 10 Ω resistor, both of which are in parallel with a 10 Ω resistor, all of these are in series with a 5 V power supply.



a. Determine the inductance, L, of the solenoid.

Ans. $L = \mu_o N^2 A = 3.96\text{x}10^{-6}\text{H}$

b. When the power supply is initially connected. What is the current across the solenoid?

Ans. $i_{o\text{-solenoid}} = 0\text{A}$

c. What is the initial current drawn from the power supply?

Ans. $i_o = \dfrac{V}{R} = 0.50\text{ A}$

d. After 1 minute, what is the current through the solenoid?

Ans. $i_{solenoid} = i_{max}\left(1-e^{-\frac{Rt}{L}}\right) = \left(\dfrac{5\text{V}}{10\Omega}\right)\left(1-e^{-\frac{60\text{s}}{3.96\text{x}10^{-7}\text{s}}}\right) = 0.50\text{ A}$

e. What is the total current drawn from the power supply?

Ans. $i_{tot} = 1.00\text{A}$

f. How much energy is stored in the inductor after 1 minute?

Ans. $U = \frac{1}{2}Li^2 = 4.95\text{x}10^{-7}\text{ J}$