## Magnetic Fields & Force

1. A point charge, q= $5\times10^{-6}$ C and m=$1\times10^{-3}$kg, travels with a velocity of: $\vec{v} = 30\frac{m}{s}\ \hat{i}$ then enters a magnetic field: $\vec{B} = 1\times10^{-6}\text{T}\ \hat{j}$.

a. What is the kinetic energy of the point charge?

Ans. $K = \frac{1}{2}mv^2 = 0.45\ \text{J}$

b. What is the magnitude of the magnetic force that acts on the charge once it has entered the field?

Ans. $\left|\vec{F}_B\right| = qvB = 1.5\times10^{-10}\ \text{N}$

c. What is the magnetic force <u>vector</u> exerted on the charge just as it enters the field?

Ans. $\vec{F}_B = q\vec{v}\times\vec{B} = 1.5\times10^{-10}\text{N}\ \hat{k}$

d. Why does the magnetic force exerted on the point charge not change its kinetic energy?

Ans. The magnetic force is always to the direction of travel.

2. Initially at rest, a charged particle, q=$+1.6\times10^{-19}$ C and m=$1.67\times10^{-27}$ kg, is accelerated through a region of constant electric field ($\vec{E} = E_y\hat{j}$), across a potential difference of V = 100V. The charged particle then enters a magnetic field: $\vec{B} = 10^{-3}\ \text{T}\ \hat{i}$.

a. What is the kinetic energy of the particle just as it enters the magnetic field? *Apply the Conservation of Energy to the particle.*

Ans. $K = \Delta K = \Delta U = q\Delta V = 1.6\times10^{-17}\ \text{J}$

b. Determine the magnetic force vector exerted on the charge, in component form, as it enters the field?

Ans. $v = \sqrt{\frac{2K}{m}} = 1.38\times10^{5}\ \frac{m}{s} \Rightarrow \vec{F}_B = q\vec{v}\times\vec{B} = -2.21\times10^{-17}\text{N}\ \hat{k}$

c. In which direction is the particle deflected once it enters the field? Calculate the radius of the particles path.

Ans. Clockwise

3. A point charge, $q=1\text{x}10^{-6}$ C and m=, travels with a velocity of:
   $\vec{v} = 30\frac{m}{s}\ \hat{i} + 50\frac{m}{s}\ \hat{j} + 40\frac{m}{s}\ \hat{k}$ , in a magnetic field: $\vec{B} = 0.01\text{T}\ \hat{j} - 0.05\text{T}\ \hat{k}$.

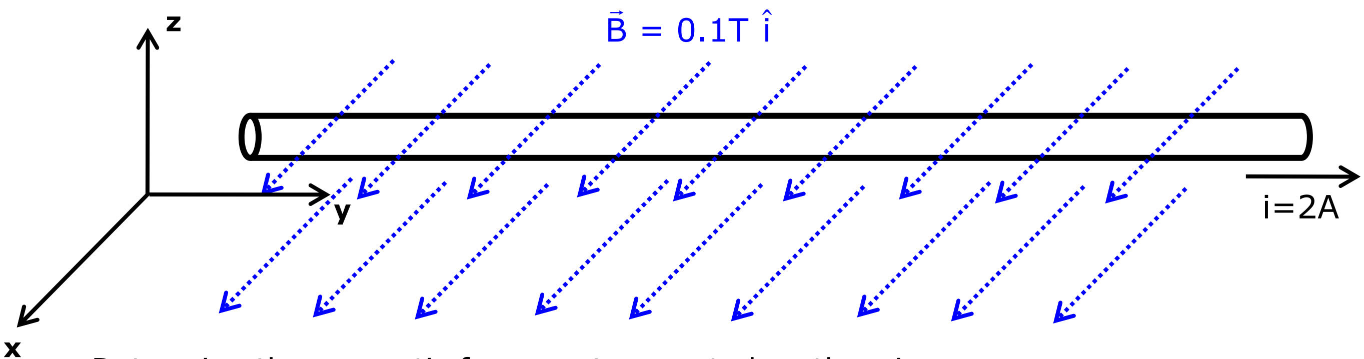
a. Determine the magnetic force vector exerted on the charge, in component form, as it enters the field?

Ans. $\vec{F}_B=q\vec{v}\times\vec{B} =(v_yB_z-B_yv_z)\hat{i}+(v_zB_x-B_zv_x)\hat{j}+(v_xB_y-B_xv_y)\hat{k}=(2.9\text{N})\hat{i}+(1.5\text{N})\hat{j}+(0.3\text{N})\hat{k}$

b. What is the magnitude of the magnetic force vector?

Ans. $\left|\vec{F}_B\right|=\sqrt{(2.9\text{N})^2+(1.5\text{N})^2+(0.3\text{N})^2}\ = 3.28\text{N}$

4. A current carrying copper wire, i=2A and length L=1.5 m, oriented along the y direction is positioned in a constant magnetic field, as shown below:




a. Determine the magnetic force vector exerted on the wire.

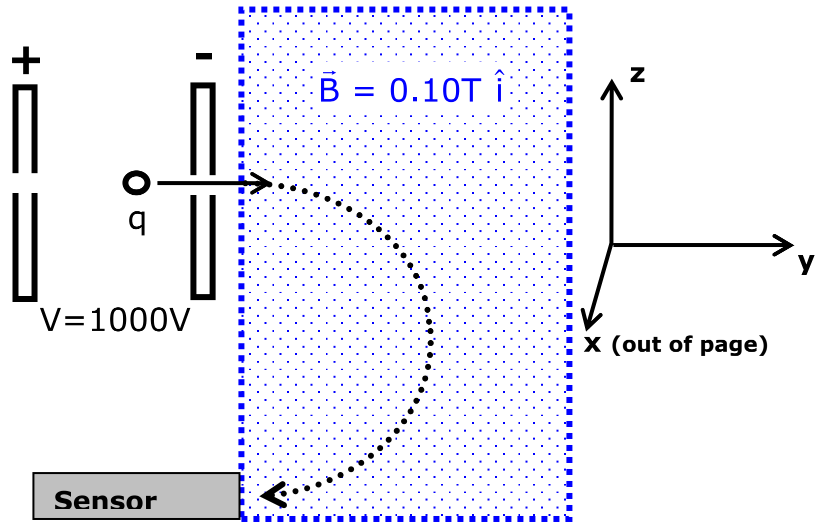
Ans. $\vec{F}_B=i\vec{L}\times\vec{B} =-0.3\text{N}\ \hat{k}$

b. Verify that the relation $\vec{F}_B=i\vec{L}\times\vec{B}$ is equivalent to $\vec{F}_B=Nq\vec{v}\times\vec{B}$, where N is the number of charge carriers in the wire affected by the magnetic field, i.e. iL = Nqv.

Ans. $\vec{F}_B=i\vec{L}\times\vec{B} =\frac{d}{dt}\left[Ne\vec{L}\times\vec{B}\right]= Ne\left(\frac{d\vec{L}}{dt}\times\vec{B}\right)\ = q\vec{v}\times\vec{B}$

## Mass Spectrometer

5. A mass spectrometer is used to measure the mass of charged particles. Initially at rest, a beam of charged particles, each with $q=+1.6\text{x}10^{-19}$ C and $m= 1.67\text{x}10^{-27}$kg, is accelerated through a small aperture across a charged capacitor (V = 1000V), following which the particle then enters a magnetic field.




a. Determine the magnetic force vector exerted on the charge, in component form, as it enters the field?

Ans.

$$v = \sqrt{\frac{2q\Delta V}{m}} = 4.38\text{x}10^5\ \tfrac{m}{s}$$

$$\vec{F}_B = q\vec{v}\times\vec{B} = -7.00\text{x}10^{-15}\text{N}\ \hat{k}$$

b. The particle completes a semicircular path within the B field and its displacement is measured with a sensor. How far from the entry point does the particle reach the sensor?

Ans.

$$\left|\vec{F}_B\right| = \frac{mv^2}{r} = 7.00\text{x}10^{-15}\text{N}\ \hat{k}$$

$$d = 2r = \frac{2mv^2}{\left|\vec{F}_B\right|} = \frac{4K}{\left|\vec{F}_B\right|}$$

$$d = 0.0914\ \text{m}$$

c. A second particle, same charge, is then measured with the device. In order for this particle to reach the sensor (same position as above), the B field is increased to 0.62T. What is the mass of the particle?

Ans. $\frac{m_1}{m_2} = \frac{B_1^2}{B_2^2} \Rightarrow m_2 = m_1\frac{B_2^2}{B_1^2} = 6.42\text{x}10^{-26}\text{kg}$

d. An important experiment in development of electromagnetism was the measurement of the mass to charge ratio for an electron, $m_e/e$. Derive an equation for the ratio of electron mass to charge in terms of r, B, V using this mass spectrometer.

Ans.

$$\frac{mv^2}{r} = evB \Rightarrow \frac{m}{e} = \frac{Br}{v} \quad \text{since} \quad eV = \tfrac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2eV}{m}}$$

$$\frac{m}{e} = Br\sqrt{\frac{m}{2eV}} \Rightarrow \frac{m}{e} = \frac{B^2r^2}{2V}$$

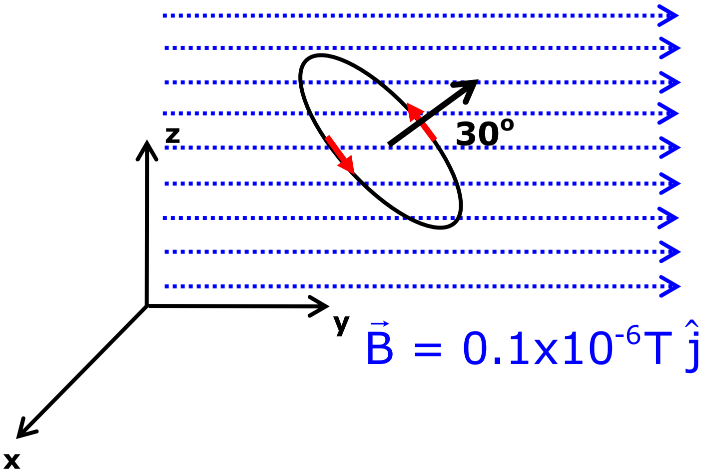
e. What magnetic field strength would be needed to direct the electron's path to the sensor?

Ans. $\frac{m_e}{m_p}=\frac{B_2^2}{B_1^2} \Rightarrow B_1=B_2\sqrt{\frac{m_e}{m_p}} = 0.0023\text{ T}$

f. What is the electric field vector, in component form, that must be applied to the particle beam, while in the magnetic field, that keep it from deflecting so that it travels directly to the opposite side?

Ans.
$$e\vec{E} = e\vec{v}\times\vec{B} \Rightarrow \vec{E} = \vec{v}\times\vec{B}$$
$$\vec{E} = vB\,\hat{k}$$

6. A circular loop of wire of radius 0.5 m, area vector $\vec{A} = A_y\hat{j} + A_z\hat{k}$, and current i=3A (counterclockwise as viewed from the right), is positioned within a constant magnetic field, as shown in the figure below. *The angle between the area vector and the field is 30° in the y-z plane.*




a. What is the magnetic moment for the current loop?

Ans.
$$\vec{\mu} = NiA(\cos30^o\hat{j} + \sin30^o\hat{k})$$
$$\vec{\mu} = \left(2.04\text{ A}\cdot\text{m}^2\right)\hat{j} + \left(1.18\text{ A}\cdot\text{m}^2\right)\hat{k}$$

b. Determine the torque vector exerted on the current loop.

Ans.
$$\vec{\tau} = \vec{\mu}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2.04\text{ A}\cdot\text{m}^2 & 1.18\text{ A}\cdot\text{m}^2 \\ 0 & 0.1\text{x}10^{-6}\text{T} & 0 \end{vmatrix}$$
$$\vec{\tau} = -1.18\text{x}10^{-7}\text{N}\cdot\text{m }\hat{i}$$

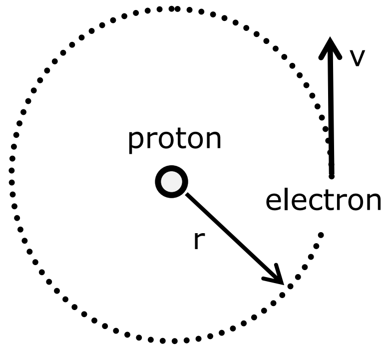
c. What is the magnitude and direction of the torque vector?

Ans. $|\vec{\tau}| = 1.18\text{x}10^{-7}\text{N}\cdot\text{m}$ in the -x direction, this will produce clockwise rotation

d. For a net torque vector of magnitude 5.0 N·m, determine the radius of the loop.

Ans.
$$|\vec{\mu}| = Ni(\pi r^2) = \frac{|\vec{\tau}|}{|\vec{B}|\sin30^o}$$
$$r = \sqrt{\frac{|\vec{\tau}|}{Ni\pi|\vec{B}|\sin30^o}} = 3.26\text{x}10^3\text{N}\cdot\text{m}$$

7. In the Bohr Model of the hydrogen atom, the electron moves around the proton at a speed of $2.2 \times 10^6$ m/s in a circle of radius $5.3 \times 10^{-11}$ m.




a. Treat the orbiting electron as a small current loop. Determine the magnetic moment associated with this motion. *{Hint: The electron travels around the circle in a time equal to the period of motion.}*

Ans. $|\vec{\mu}| = iA = \frac{e}{T}A = \frac{e\pi r^2}{\left(\frac{2\pi r}{v}\right)} = 9.33\text{x}10^{-24}\,\text{A}\cdot\text{m}^2$

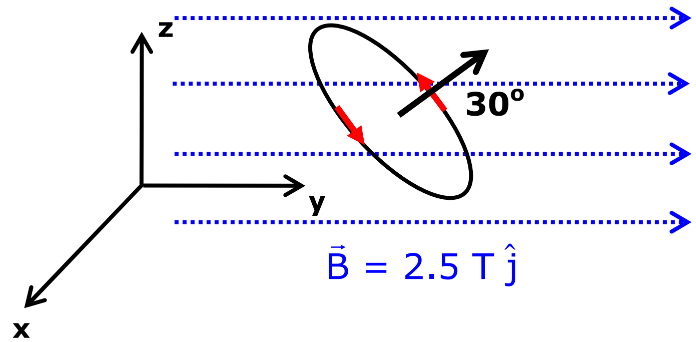
b. Determine the magnitude of the magnetic field at the center of the electron's orbit.

Ans. using equation 29.9 from Ch29: $|\vec{B}| = \frac{\mu_o e}{2Tr} = \frac{\mu_o e}{2\left(\frac{2\pi r}{v}\right)r} = \frac{\mu_o ev}{4\pi r^2} = 1.26\text{x}10^3\,\text{T}$

c. This atom is placed in a constant magnetic field directed initially 30° with respect to the area vector of the circular orbit of the electron. What is the magnitude and direction of the momentary torque vector exerted on the electron by this magnetic field? *Assume the area vector is in the y-z plane.*

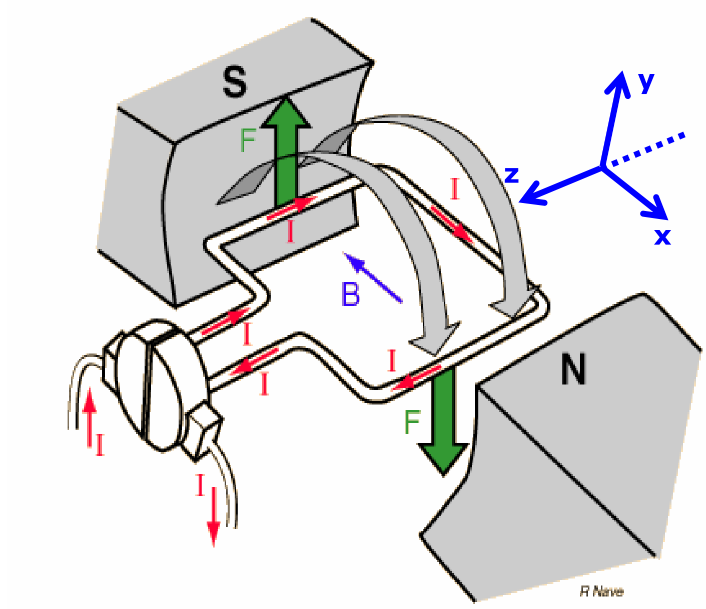
Ans. $\vec{\tau} = \vec{\mu}\times\vec{B} = -\mu\cdot B\cdot\sin 30^\circ\ \hat{i}$

$\vec{\tau} = -1.17\text{x}10^{-23}\,\text{N}\cdot\text{m}\ \hat{i}$



**DC Motor**

8. A 2.5A current flows through the rotating wire loop, called an armature, of a DC motor, as shown in the figure. The dimensions of the armature are 0.2m by 0.2 m. The constant magnetic field between the magnets is has a magnitude of $1.0\text{x}10^{-4}$ T.




a. Determine the magnetic force on either of the armature segments that is parallel to the magnetic field.

Ans. $\left|\vec{F}_B\right| = i\left|\vec{L}\times\vec{B}\right| = iLB = 5.0\text{x}10^{-5}\text{N}$

b. At what rotational position of the armature is the force exerted on the wire maximum? What about the torque?

Ans. Torque is maximum when $\vec{A}$ is $\perp$ to $\vec{B}$

c. Calculate the torque exerted on the armature for the figure shown.

Ans.

$$\left|\vec{\tau}\right| = \left|\vec{r}_1\times\vec{F}_1\right| + \left|\vec{r}_2\times\vec{F}_2\right| = 2\left|\frac{\vec{L}}{2}\times\vec{F}_B\right| = 2\left(\frac{0.2\text{ m}}{2}\right)\left(5.0\text{x}10^{-5}\text{N}\right) = 1.0\text{x}10^{-5}\text{N}\cdot\text{m}$$

d. Derive the general equation for the torque exerted on the armature. *Assume that when the armature rotates 180°, the direction of the electric current instantaneously reverses direction to that it never resists the direction of the rotation.*

Ans. $\left|\vec{\tau}(t)\right| = iL^2B\left|\sin(\omega t+\phi)\right| = iL^2B\left|\sin\theta\right|$

e. Estimate the average torque generated by the DC motor when it is in operation.

Ans.

$$\left|\vec{\tau}_{avg}\right| = \frac{iL^2B\int_0^{\pi}\sin\theta\, d\theta}{\int_0^{\pi}d\theta} = -\frac{iL^2B}{\pi}\cos\theta\Big|_0^{\pi} = \frac{2iL^2B}{\pi} = 6.37\text{x}10^{-6}\text{N}\cdot\text{m}$$

f. When the number of loops is 100, what is the average torque generated by the DC motor while it is in operation?

Ans.

$$\left|\vec{\tau}_{avg}\right| = N\left|\vec{\tau}_{avg_{N=1}}\right| = 100(6.37\text{x}10^{-6}\text{N}\cdot\text{m}) = 6.37\text{x}10^{-4}\text{N}\cdot\text{m}$$