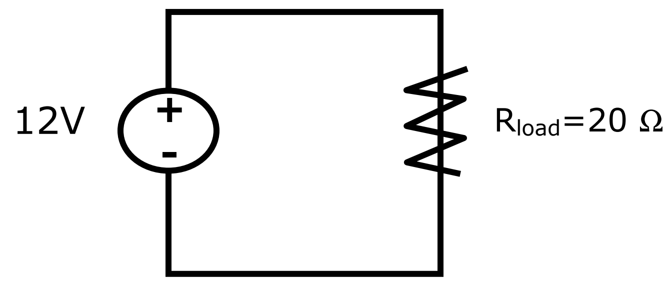
Internal Resistance

1. A real battery has a potential difference of 12V when no load is attached. A 20 Ω load circuit is then attached to the battery, drawing 5 W of power.



   a. What is the current that flows through the load circuit?

Ans. $i = \sqrt{\dfrac{P}{R_{load}}} = \sqrt{\dfrac{5W}{20\Omega}} = 0.5A$

   b. What is the potential difference across the load circuit?

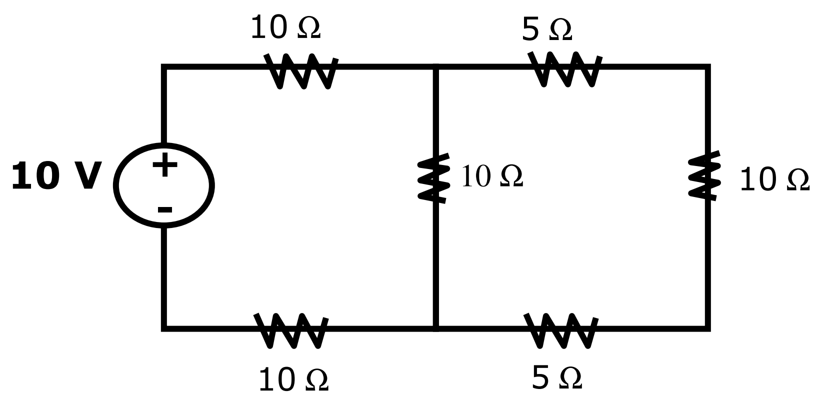
Ans. $V_{load} = iR = 10V$

   c. What is the internal resistance of the battery?

Ans. $R_{internal} = \dfrac{V\text{-}V_{output}}{i_{load}} = \dfrac{2V}{0.5\ A} = 4\Omega$

Ohm's Law & Electrical Circuits:

2. Consider the following circuit:



   a. What is the equivalent resistance of this circuit?

Ans. $R_{eq} = 20\Omega + \dfrac{1}{\dfrac{1}{10\Omega} + \dfrac{1}{20\Omega}} = 26.7\Omega$

   b. How much total current is drawn from the battery?

Ans. $i = \dfrac{10V}{26.7\Omega} = 0.375A$

   c. How much total power is drawn from the power source?

Ans. $P = Vi = i^2R = \dfrac{V^2}{R} = 3.75W$

   d. If the circuit is connected for 10 minutes, how much energy is drawn from the battery?

Ans. $E = Pt = (3.75W)(600s) = 2250J$

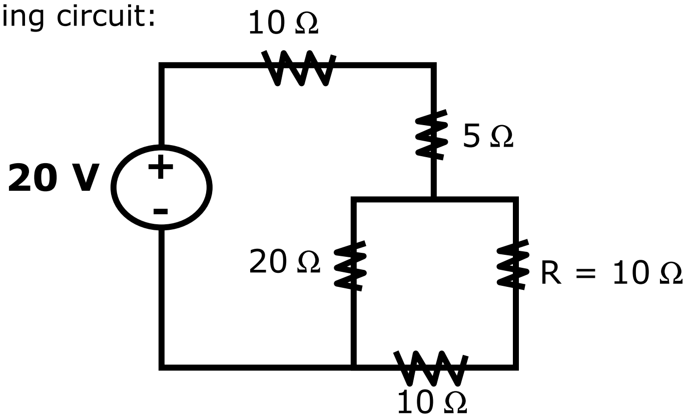
   e. How much charge?

Ans. $q = it = (0.375A)(600s) = 225\ C$

   f. How many electrons?

Ans. $N = \dfrac{q}{e} = \dfrac{225\ C}{1.6x10^{-19}C} = 1.41x10^{21}\ e^{-}s$

3. Consider the following circuit:



a. What is the equivalent resistance of this circuit?

Ans. $R_{eq}=15\Omega+\dfrac{1}{\dfrac{1}{20\Omega}+\dfrac{1}{20\Omega}}=25\Omega$

b. How much total current is drawn from the battery?

Ans. $i=\dfrac{20V}{25\Omega}=0.8A$

c. How much current is flowing through each resistor?

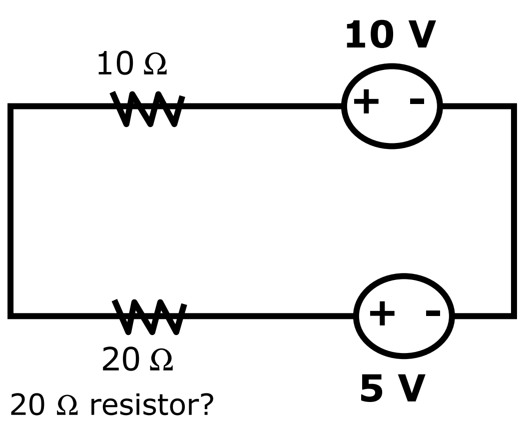
Ans. The current through the first 2 resistors: i = 0.8A

The current through each of the resistors in parallel: i = 0.4A

d. How much power is dissipated across resistor R?

Ans. $P=i^2R=(0.4A)^2(10\Omega)=1.6W$

4. Consider the following circuit:



a. What is the current through the 20 Ω resistor?

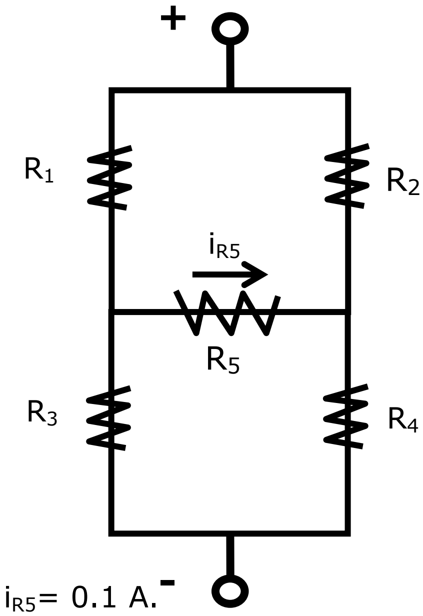
Ans. Loop Rule: $10V-i(10\Omega)-i(20\Omega)-5V=0 \Rightarrow i=\dfrac{5V}{30\Omega}=0.17A$ (counter-clockwise)

b. What is the current through the 10 Ω resistor?

Ans. Same as through the 20Ω resistor, i=0.74A

5. A "Wheatstone Bridge" circuit is shown in the following diagram. The voltage from the top terminal to the bottom terminal is 120 V and the resistor values are:

R_1=20 Ω R_2=??
R_3=50 Ω R_4=40 Ω
R_5=10 Ω



a. Determine the value of R_2 when the current through R_5 is zero, (i_{R5}= 0 A).

Ans. $R_2=R_4(R_1/R_3) = 16\Omega$

b. What is the equivalent resistance for this circuit?

Ans. $R_{eq}=31.1\Omega$

c. What is the current through R_2?

Ans. $i=(120V)/(56\Omega)=2.14A$

d. Calculate the rate of resistive heat loss (power) through R_2.

Ans. $P=i^2R=(2.14A)^2(16\Omega)=73.3W$

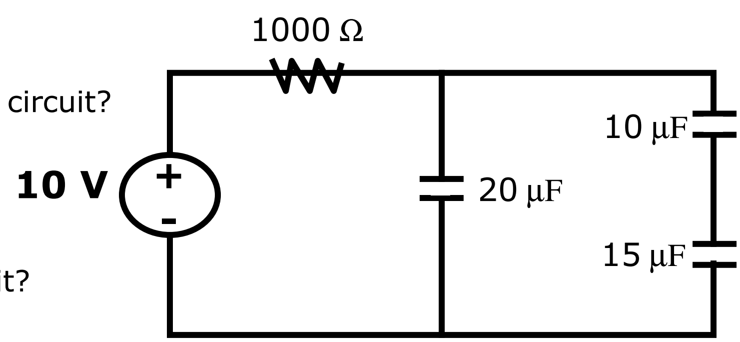
e. Determine the value of R_2 when the current through R_5 is zero, i_{R5}= 0.1 A.

Ans. When $i_{R5}=0.1A$, $\Delta V=i_{R5}R_5=(0.1A)(10\Omega)=1V$

Note the following relations:
A) $i_1R_1+i_3R_3=120V$ & $i_2R_2+i_4R_4=120V$
B) $i_1R_1-i_2R_2=1V$ & $i_4R_4+i_3R_3=1V$
C) $i_1-i_3=0.1A$ & $i_4-i_2=0.1A$

Solving for $R_2 \rightarrow R_2 = 17.1\ \Omega$

6. Consider the following RC circuit.



a) What is the equivalent capacitance for this circuit?

Ans. $$C_{eq}= 20\mu F+\frac{1}{\frac{1}{10\mu F}+\frac{1}{15\mu F}}=26\mu F$$

b) What is the RC time constant for this circuit?

Ans. RC=0.026s

c) What is the significance of this RC constant?

Ans. For every time interval, t=RC=0.026s, a discharging capacitor will lose 63.2% of its initial charge (at the beginning of the interval) and a charging capacitor will gain 63.2% of its initial remaining charge (at the beginning of the time interval

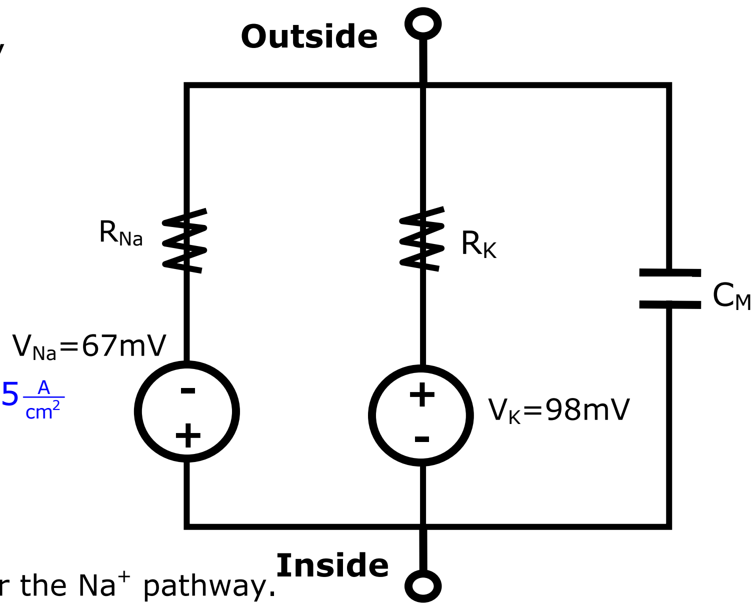
d) If the battery is removed from the circuit and the capacitors are allowed to discharge, how long would it take for the charge in the capacitor to be 20% of the fully charged value?

Ans. $$t=RC\cdot\ln\left(\frac{q_{max}}{q}\right)=(0.026s)\ln\left(\frac{1}{0.20}\right)=0.42s$$

e) What would be the potential difference across the discharged capacitor in (d)?

Ans. $$V = V_{max}\left(\frac{q}{q_{max}}\right)=(10V)(0.20)=2.0V$$

7. The Hodgkin-Huxley model of the giant squid axon surface membrane is illustrated below. In this model, you will consider 2 independent and exclusive ion pathways, Na^+ and K^+, respectively. The ions can only flow through their corresponding pathway. The resting (equilibrium) membrane potential ($\Delta V=V_{inside}-V_{outside}$) is -65 mV and the conductance per unit area of the K^+ pathway is 50 $m\Omega^{-1}/cm^2$.



a. What is the direction of Na^+ and K^+ ion flow, respectively?

Ans. K^+ flows outward, Na^+ flows inward

b. What is the net current density across the surface membrane at equilibrium?

Ans. At equilibrium, i_{net} = 0 so J_{net}=0

c. What is the K^+ current density?

Ans. $$J_{K^+}=\left(\frac{50\text{x}10^{-3}\Omega^{-1}}{cm^2}\right)(33\text{x}10^{-3}V)=0.00165\frac{A}{cm^2}$$

d. What is the Na^+ current density?

Ans. $J_{Na}=J_K=0.00165A/cm^2$ (inward)

e. Determine the conductance per unit area for the Na^+ pathway.

Ans. $$\frac{g_{Na^+}}{A}=\frac{J_{Na^+}}{V+V_{Na^+}}=\frac{0.00165\frac{A}{cm^2}}{2\text{x}10^{-3}V}=0.0066\frac{\Omega^{-1}}{cm^2}$$

f. Determine the resistance per unit area for each ion, R_{Na}/A and R_K/A.

Ans.

$$\frac{R_{K^+}}{A}=\frac{\left(\frac{1}{50\text{x}10^{-3}\Omega^{-1}}\right)}{cm^2}=20.\frac{\Omega}{cm^2}$$

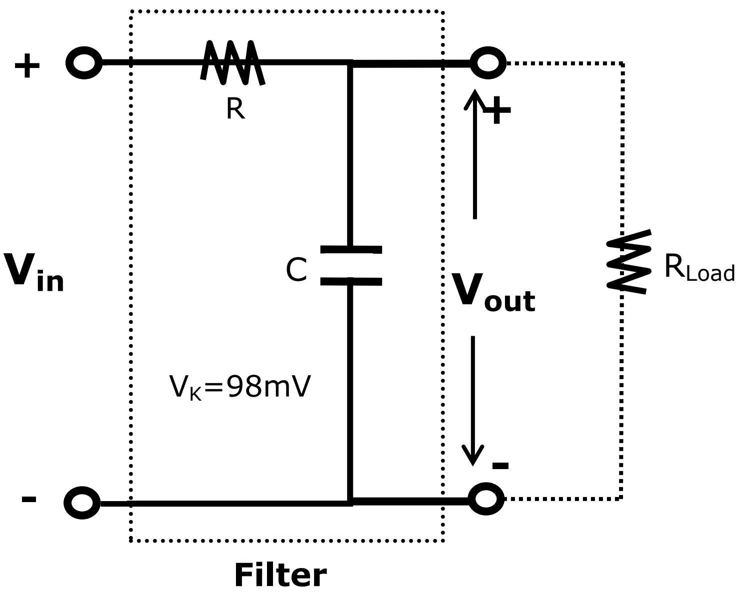
$$\frac{R_{Na^+}}{A}=\frac{\left(\frac{1}{6.6\text{x}10^{-3}\Omega^{-1}}\right)}{cm^2}=150\frac{\Omega}{cm^2}$$

g. What is the charge density of the surface membrane? *Hint: Treat the surface membrane as a capacitor, where the capacitance per unit surface area is 0.01 F/m².*

Ans. $$\frac{q}{A}=V\left(\frac{C}{A}\right)=(65\text{x}10^{-3}V)\left(0.01\frac{F}{m^2}\right)=6.5\text{x}10^{-4}\frac{C}{m^2}=6.5\text{x}10^{-8}\frac{C}{cm^2}$$

Application: The Low Pass Filter

8. A simple filter circuit can be constructed from a resistor in series with a capacitor. An input voltage source is connected to the left side terminals and a "load circuit" is connected to the output terminals. An advantage of this circuit design is that it can resist current spikes associated with unexpected, rapid voltage jumps, which would otherwise damage the load circuit.



a. What is the ratio of the input to output voltage, in terms of the filter resistance R and R_{load}?

Ans. $\dfrac{V_{out}}{V_{in}} = \dfrac{R_{Load}}{R + R_{Load}} = 0.833$

b. What is the necessary DC input voltage to achieve a desired output voltage of 10 V? Assume that R is 10Ω and R_{load} is 50Ω.

Ans. $V_{in} = \dfrac{V_{out}\left(R + R_{Load}\right)}{R_{Load}} = 12.0\text{V}$

c. When the filter capacitor is completely charged, how much current flows into the load circuit?

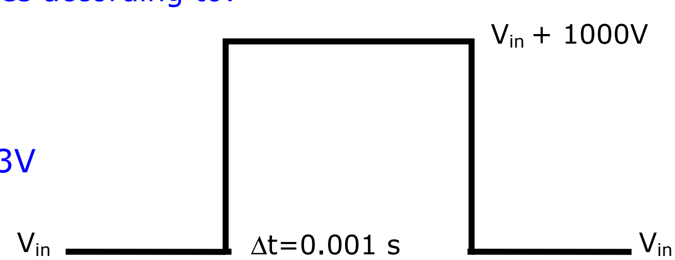
Ans. $i = i_{Load} = \dfrac{V_{in}}{R + R_{Load}} = \dfrac{V_{out}}{R_{Load}} = 0.20\text{A}$

d. For a voltage spike (square wave) that increases the input voltage by 1000V for a time of 0.001 s, what is the final voltage across the filter capacitor (5.0 mF) at the top of the voltage spike?

Ans. The voltage across the capacitor rises as it charges according to:

$$V_{out}(t) = V_c(t) = \left(\frac{\Delta V_{in}}{R}\right)\left(1-e^{-\frac{t}{RC}}\right) + V_{out_{initial}}$$

$$V_{out}(t) = V_c(t) = \left(\frac{1000\text{V}}{60\Omega}\right)\left(1-e^{-\frac{0.001\text{s}}{60\text{s}}}\right) + 10\text{V} = 10.33\text{V}$$



e. What is the peak current flow into the capacitor?

Ans. The peak current occurs immediately as with the voltage spike (t=0s):

$$i_{peak} = \frac{\Delta V_{in}}{R} = 100.\ \text{A}$$

f. What is the peak current flow into the load circuit?

Ans. The peak load current occurs at the end of the voltage spike (t=0.001 s):

$$i_{load\text{-}peak} = \frac{V_{out}}{R_{load}} = \left(\frac{\Delta V_{in}}{R_{load}+R}\right)\left(1-e^{-\frac{t}{RC}}\right) + \frac{V_{out_o}}{R}$$

$$i_{load\text{-}peak} \cong 0.53\ \text{A}$$