

Ohm's Law & Electrical Circuits:

1. Electric Current flows through the following network:

a. What is the magnitude and direction of the current, i ?

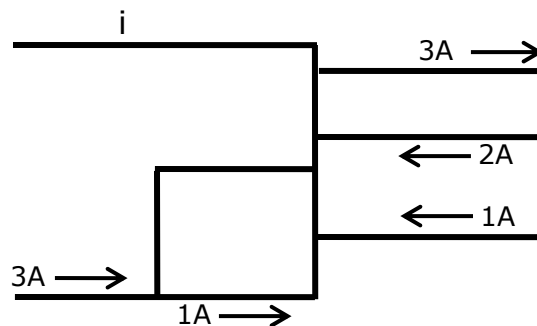
Ans. $i_{\text{Net}} = 3A + i = 0 \rightarrow i = -3A$ or $3A$ (to the left)

b. How many coulombs of charge flow in 2.0 min, for constant i ?

Ans. $q = it = (3 \text{ C/s})(120\text{s}) = 360\text{C}$ (to the left)

c. How many electrons flow in 2.0 min, for constant i ?

Ans. $N = q/e = (-360\text{C})/(-1.6 \times 10^{-19}\text{C}/e^-) = 2.25 \times 10^{21} e^- \text{'s}$ (to the right)



2. Electric Current flows through the following network:

a. What is the magnitude and direction of the current, i ?

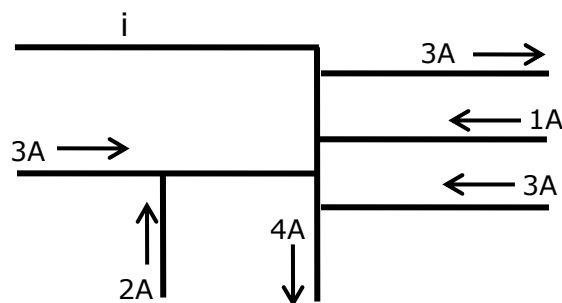
Ans. $i_{\text{Net}} = 3A + 2A + 1A + 3A - 3A - 4A + i = 0$
 $\rightarrow i = -2A$ or $2A$ (to the left)

b. How much charge flows in 5.0 min, for constant i ?

Ans. $q = it = (2 \text{ C/s})(300\text{s}) = 600\text{C}$ (to the left)

c. How many electrons flow in 5.0 min, for constant i ?

Ans. $N = q/e = (-600\text{C})/(-1.6 \times 10^{-19}\text{C}/e^-) = 3.75 \times 10^{21} e^- \text{'s}$ (to the right)



3. Starting at $t_0 = 0\text{s}$, a wire conducts a time-changing current, $i(t)$ of the form: $i(t) = i_{\text{max}} e^{-\frac{t}{\tau}}$, where $i_{\text{max}} = 5.0 \text{ A}$ and $\tau = 0.5 \text{ s}$. The resistance in the wire is 5Ω .

a. How much charge, in coulombs, will flow along the wire in 1.5 s?

Ans. $q(t) = \int_0^t i_{\text{max}} e^{-\frac{t}{\tau}} dt = -(i_{\text{max}} \tau) e^{-\frac{t}{\tau}} \Big|_0^t = (i_{\text{max}} \tau) (1 - e^{-\frac{t}{\tau}})$
 $q(t) = (5.0\text{A})(0.5\text{s}) (1 - e^{-\frac{1.5\text{s}}{0.5\text{s}}}) = 2.4\text{C}$

b. How many electrons flow along the wire in 1.5s?

Ans. $N = q/e = (2.4\text{C})/(1.6 \times 10^{-19}\text{C}/e^-) = 1.5 \times 10^{19} e^- \text{'s}$

c. What is the equation for the voltage as a function of time across this wire?

Ans. $V(t) = i(t)R = (i_{\text{max}} R) e^{-\frac{t}{\tau}} = (25\text{V}) e^{-\frac{t}{\tau}}$

4. A resistor has cylindrical geometry, with a resistivity $\rho = 3.5 \times 10^{-5} \Omega \cdot \text{m}$ at 20°C and temperature coefficient $\alpha = -5.0 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$. The length of the resistor is $1.0 \times 10^{-2} \text{ m}$ and the radius is $1.0 \times 10^{-4} \text{ m}$.

a. What is the cross sectional area of the wire?

Ans. $A = \pi r^2 = 3.1 \times 10^{-8} \text{ m}^2$

b. Determine the resistance at 20°C .

Ans. $R = \rho \frac{L}{A} = 11 \Omega$

c. What is the resistivity and resistance at 150°C ?

Ans. $\rho = \rho_0(1 + \alpha \Delta T) = 3.3 \times 10^{-5} \Omega \cdot \text{m}$
 $R = R_0(1 + \alpha \Delta T) = 10 \Omega$

5. Consider the electrons of a car battery traveling along a wire toward a starting motor. Assume that a current of 300 A travels along copper wire (diameter = 0.3 cm and length = 0.85 m) and the # of charge carriers is $8.49 \times 10^{28} \text{ m}^{-3}$.

a. Determine the cross sectional area of the wire.

Ans. $A = \pi r^2 = 7.1 \times 10^{-6} \text{ m}^2$

b. What is the drift velocity of the electrons in this current?

Ans. $v_{\text{drift}} = \frac{J}{ne} = \frac{i}{Ane} = 0.0031 \frac{\text{m}}{\text{s}}$

c. How long will it take electrons to get from a car battery to the motor?

Ans. $t = \frac{L}{v_{\text{drift}}} = \frac{0.85 \text{ m}}{0.0031 \frac{\text{m}}{\text{s}}} = 270 \text{ s}$

6. The bulb of a typical light bulb is conducts 0.3 A of current at 2.9 V. At room temperature, 20°C , the resistance of the tungsten filament is 1.1Ω .

a. What is the resistance of the bulb filament at 2.9 V?

Ans. $R = \frac{V}{i} = \frac{2.9 \text{ V}}{0.3 \text{ A}} = 9.7 \Omega$

b. What is the temperature of the illuminated tungsten filament?

Ans. $T = \frac{\left(\frac{R}{R_0} - 1 \right)}{\alpha} + 20^\circ\text{C} = 1750^\circ\text{C}$

c. How thick is the tungsten filament if the length of filament is 2.0 cm?

Ans. $d = 2r = 2\sqrt{\frac{\rho L}{\pi R}} = 3.6 \times 10^{-5} \text{ m}$

7. Estimate the temperature where lead becomes a superconductor.

$$\text{Ans. } \rho = \rho_0(1 + \alpha\Delta T) = 0 \Rightarrow T = -\frac{1}{\alpha} + 20^\circ\text{C} = -236^\circ\text{C} \text{ (37K)}$$

8. Estimate the temperature that copper is predicted to become a superconductor.

$$\text{Ans. } \rho = \rho_0(1 + \alpha\Delta T) = 0 \Rightarrow T = -\frac{1}{\alpha} + 20^\circ\text{C} = -127^\circ\text{C} \text{ (146K)}$$

9. Estimate the temperature that silicon is predicted to become a superconductor.

$$\text{Ans. } \rho = \rho_0(1 + \alpha\Delta T) = 0 \Rightarrow T = -\frac{1}{\alpha} + 20^\circ\text{C} = 162^\circ\text{C} \text{ (436K)}$$

10. An electrical cable consists of 125 strands, each with a resistance of $2.65 \mu\Omega$. A constant potential difference is applied across the cable resulting in current of 0.75 A along the cable.

a. What is the current in each strand?

$$\text{Ans. } i_{\text{strand}} = \frac{i_{\text{tot}}}{125} = 0.006\text{A}$$

b. What is the applied potential difference?

$$\text{Ans. } V = i_{\text{strand}} R_{\text{strand}} = 1.59 \times 10^{-8} \text{V}$$

c. What is the resistance of the cable?

$$\text{Ans. } R_{\text{cable}} = \frac{V}{i_{\text{tot}}} = 2.1 \times 10^{-8} \Omega$$

11. A certain space heater, operating at 120V, dissipates heat energy at a rate of 500 W during operation.

a. What is the resistance of the heater during operation?

$$\text{Ans. } R = \frac{P}{V^2} = 29 \Omega$$

b. How much current flows through the heating element during operation?

$$\text{Ans. } i = \frac{P}{V} = 4.2\text{A}$$

c. At what rate do electrons flow through any given cross section of the heater element?

$$\text{Ans. } \frac{dN_e}{dt} = \frac{i}{e} = \frac{4.2 \frac{\text{C}}{\text{s}}}{1.6 \times 10^{-19} \frac{\text{C}}{e^-}} = 2.6 \times 10^{19} \frac{e^-}{\text{s}}$$

d. What is the heat energy transfer per coulomb?

$$\text{Ans. } \frac{Q}{q} = \frac{P}{i} = \frac{500 \frac{\text{J}}{\text{s}}}{4.2 \frac{\text{C}}{\text{s}}} = 120 \frac{\text{J}}{\text{C}} \text{ (or V)}$$

e. What is the heat energy transfer per electron?

$$\text{Ans. } \frac{Q}{e} = (120 \frac{\text{J}}{\text{C}}) (1.6 \times 10^{-19} \frac{\text{C}}{e^-}) = 1.9 \times 10^{-17} \frac{\text{J}}{e^-}$$