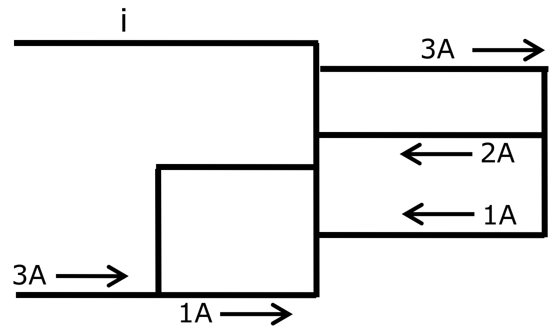
Ohm's Law & Electrical Circuits:

1. Electric Current flows through the following network:



a. What is the magnitude and direction of the current, i?

Ans. $i_{Net} = 3A + i = 0 \rightarrow i = -3A$ or 3A (to the left)

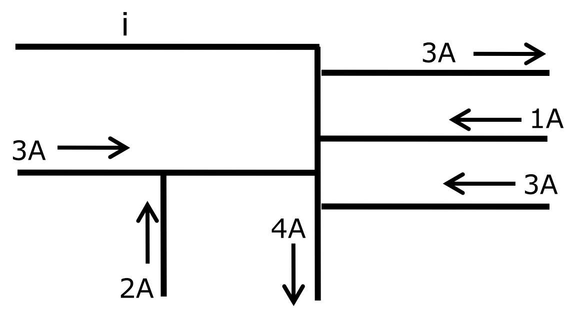
b. How many coulombs of charge flow in 2.0 min, for constant i?

Ans. q = it = (3 C/s)(120s) = 360C (to the left)

c. How many electrons flow in 2.0 min, for constant i?

Ans. $N = q/e = (-360C)/(-1.6\times10^{-19}C/e^-) = 2.25\times10^{21}$ e^-'s (to the right)

2. Electric Current flows through the following network:



a. What is the magnitude and direction of the current, i?

Ans. $i_{Net} = 3A + 2A + 1A + 3A - 3A - 4A + i = 0$

$\rightarrow i = -2A$ or 2A (to the left)

b. How many coulombs of charge flow in 5.0 min, for constant i?

Ans. q = it = (2 C/s)(300s) = 600C (to the left)

c. How many electrons flow in 5.0 min, for constant i?

Ans. $N = q/e = (-600C)/(-1.6\times10^{-19}C/e^-) = 3.75\times10^{21}$ e^-'s (to the right)

3. Starting at $t_o = 0s$, a wire conducts a time-changing current, i(t) of the form: $i(t) = i_{max}e^{-\frac{t}{\tau}}$, where $i_{max} = 5.0$ A and $\tau = 0.5$ s. The resistance in the wire is 5 Ω.

a. How much charge, in coulombs, will flow along the wire in 2.5 min?

Ans.
$$q(t) = \int_0^t i_{max}e^{\frac{-t}{\tau}}dt = -(i_{max}\tau)e^{\frac{-t}{\tau}}\Big|_0^t = (i_{max}\tau)\left(1 - e^{\frac{-t}{\tau}}\right)$$
$$q(t) = (5.0A)(0.5s)\left(1 - e^{\frac{-150s}{0.5s}}\right) = 2.5C$$

b. How many electrons flow along the wire in 2.5 min?

Ans. $N = q/e = (-2.5C)/(-1.6\times10^{-19}C/e^-) = 1.56\times10^{19}$ e^-'s

c. What is the equation for the voltage as a function of time across this wire?

Ans. $V(t) = i(t)R = (i_{max}R)e^{-\frac{t}{\tau}} = (25V)e^{-\frac{t}{\tau}}$

4. A resistor has cylindrical geometry, with a resistivity $\rho = 3.5 \times 10^{-5}\ \Omega\cdot m$ at 20°C and temperature coefficient $\alpha = -5.0\times10^{-4}\ {}^{o}C^{-1}$. The length of the resistor is 5.0×10^{-3} m and the radius is 1.3×10^{-4}m.

a. What is the cross sectional area of the wire?

Ans. $A = \pi r^2 = 5.3\times10^{-8}\ m^2$

b. Determine the resistance at 20°C.

Ans. $R = \rho\frac{L}{A} = 3.3\Omega$

c. What is the resistivity and resistance at 150°C?

Ans. $\rho = \rho_o(1+\alpha\Delta T) = 3.3\times10^{-5}\Omega\cdot m$
$R = R_o(1+\alpha\Delta T) = 3.1\Omega$

5. Consider the electrons of a car battery traveling along a wire toward a starting motor. Assume that a current of 300 A travels along copper wire (diameter = 0.3 cm and length = 0.85 m) and the # of charge carriers is $8.49\times10^{28}\ m^{-3}$.

a. Determine the cross sectional area of the wire.

Ans. $A = \pi r^2 = 7.1\times10^{-6}\ m^2$

b. What is the drift velocity of the electrons in this current?

Ans. $v_{drift} = \frac{J}{ne} = \frac{i}{Ane} = 0.0031\frac{m}{s}$

c. How long will it take electrons to get from a car battery to the motor?

Ans. $t = \frac{L}{v_{drift}} = \frac{0.85\ m}{0.0031\frac{m}{s}} = 270s$

6. The bulb of a typical light bulb is conducts 0.3 A of current at 2.9 V. At room temperature, 20°C, the resistance of the tungsten filament is 1.1 Ω.

a. What is the resistance of the bulb filament at 2.9 V?

Ans. $R = \frac{V}{i} = \frac{2.9V}{0.3A} = 9.7\Omega$

b. What is the temperature of the illuminated tungsten filament?

Ans. $T = \frac{\left(\frac{R}{R_o} - 1\right)}{\alpha} + 20^oC = 1750^oC$

c. How thick is the tungsten filament if the length of filament is 2.0 cm?

Ans. $d = 2r = 2\sqrt{\frac{\rho L}{\pi R}} = 3.6\times10^{-5}m$

7. An electrical cable consists of 125 strands, each with a resistance of 2.65 μΩ. A constant potential difference is applied between the ends of the cable resulting in a total current of 0.75 A along the cable.

a. What is the current in each strand?

Ans. $i_{strand} = \frac{i_{tot}}{125} = 0.006\text{A}$

b. What is the applied potential difference?

Ans. $V = i_{strand}R_{strand} = 1.59\text{x}10^{-8}\text{V}$

c. What is the resistance of the cable?

Ans. $R_{cable} = \frac{V}{i_{tot}} = 2.1\text{x}10^{-8}\Omega$

8. A certain space heater, operating at 120V, dissipates heat energy at a rate of 500 W during operation.

a. What is the resistance of the heater during operation?

Ans. $R = \frac{P}{V^2} = 29\ \Omega$

b. How much current flows through the heating element during operation?

Ans. $i = \frac{P}{V} = 4.2\text{A}$

c. At what rate do electrons flow through any given cross section of the heater element?

Ans. $\frac{dN_e}{dt} = \frac{i}{e} = \frac{4.2\ \frac{C}{s}}{1.6\text{x}10^{-19}\ \frac{C}{e^-}} = 2.6\text{x}10^{19}\ \frac{e^-}{s}$

d. What is the heat energy transfer per coulomb?

Ans. $\frac{Q}{q} = \frac{P}{i} = \frac{500\frac{J}{s}}{4.2\frac{C}{s}} = 120\frac{J}{C}$ (or V)

e. What is the heat energy transfer per electron?

Ans. $\frac{Q}{e} = \left(120\frac{J}{C}\right)\left(1.6\text{x}10^{-19}\ \frac{C}{e^-}\right) = 1.9\text{x}10^{-17}\ \frac{J}{e^-}$