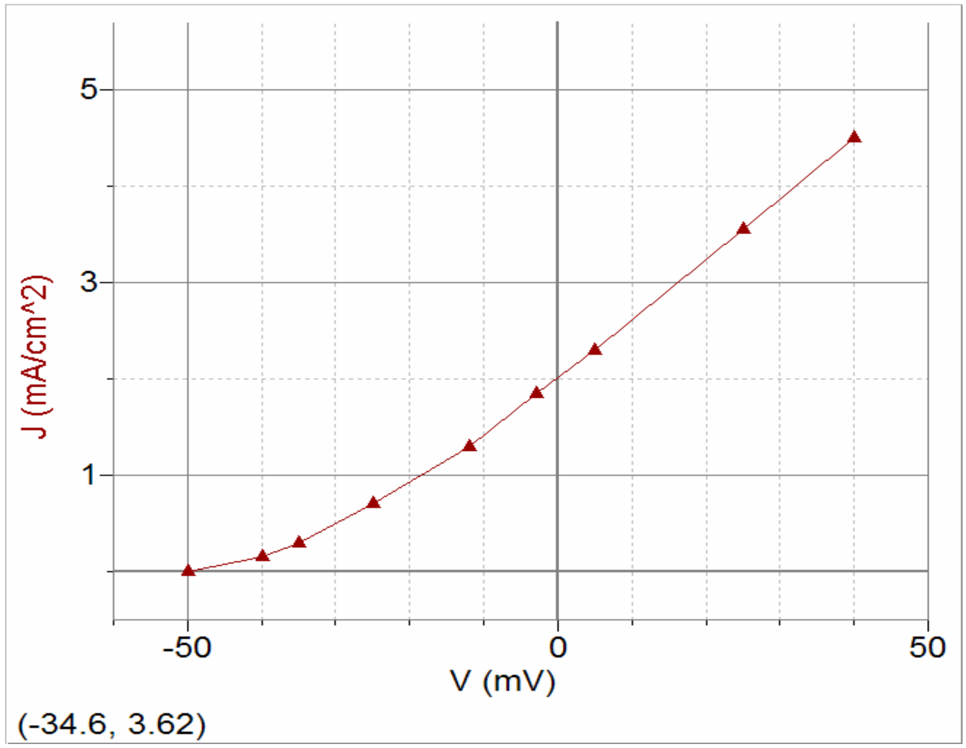
Examples in Biophysics

1. Classic biophysical studies by Hodgkin and Huxley explored the electrical properties of the giant squid (*Loligo pealei*) axon. In these studies, the current density (J_m) of K^+ (potassium ions) vs potential difference (V_m) across the axon cell membrane was measured, producing the following data points:

V_m (mV)	J_m (mA/cm^2)
-50	0.00
-40	+0.15
-30	+0.30
-25	+0.70
-12	+1.30
-3	+1.85
+5	+2.30
+25	+3.55
+40	+4.50



a. Is the surface membrane of the squid axon Ohmic (in whole or in part)? Explain.

Ans. The surface membrane is nearly Ohmic at potentials above -25 mV.

b. Estimate the slope of J_m vs V. *This is the K^+ conductance per unit area, called the specific conductance.*

Ans. The slope of J vs V, from -25 mV to +40mV:

$$\text{slope} \approx \frac{\Delta J_m}{\Delta V} = \frac{4.50\,\frac{\text{mA}}{\text{cm}^2} - 0.70\,\frac{\text{mA}}{\text{cm}^2}}{40\text{mV} - (-25\text{V})} = 0.0585\,\frac{\text{A}}{\text{Vcm}^2} \quad \left\{\text{or } 0.0585\,\frac{\Omega^{-1}}{\text{cm}^2}\right\}.$$

c. Estimate the resistance per unit area *(or the specific resistance).*

Ans. The reciprocal of the conductance per unit area is the resistance per unit area:

$$\text{spec. resistance} = \frac{R}{A} = \frac{1}{0.0585\ \Omega^{-1}\cdot\text{cm}^2} = 17.1\,\frac{\Omega}{\text{cm}^2}.$$

d. Estimate the drift velocity at +25 mV. *Assume the charge carrier density (n) is $6.0\text{x}10^{28}$ m^{-3} and charge carriers are monovalent cations (i.e. $q = +e$).*

$$\text{Ans.} \quad v_d = \frac{J}{ne} = \frac{\left(3.55\times10^{-3}\,\frac{\frac{C}{s}}{\text{cm}^2}\right)\left(\frac{1\ \text{cm}^2}{10^{-4}\ \text{m}^2}\right)}{\left(6.0\times10^{28}\text{m}^{-3}\right)\left(1.602\times10^{-19}\text{C}\right)} = 3.69\times10^{-9}\,\frac{\text{m}}{\text{s}}$$

2. A conducting protein present in the giant squid axon cell membrane, called a sodium (Na^+) ion channel, allows the flow of ion currents across the surface membrane. The conductance of a single Na^+ channel is approximately 4.0 pΩ^{-1}.

a. Estimate the resistance of a single Na^+ channel.

Ans. The reciprocal of the conductance is the resistance:

$$R = \frac{1}{g} = \frac{1}{4.0 \times 10^{-9}\ \Omega^{-1}} = 2.5 \times 10^{8}\ \Omega$$

b. Sketch the V vs i graph for the Na^+ channel. Note: the "reversal potential" (the voltage where the current is zero) in this experiment is +20 mV not 0 mV.

Ans. Assuming the sodium channel resistance is Ohmic, the graph should obey the following equation:

$$V(i) = Ri + \left(20 \times 10^{-3} V\right) = \left(2.5 \times 10^{8}\ \tfrac{V}{A}\right) i + \left(20 \times 10^{-3} V\right)$$

c. Determine how much current should flow through this channel when the applied voltage across it is +40 mV.

Ans. Substituting $40x10^{-3}$ V into the above equation:

$$i = \frac{40 \times 10^{-3} V - 20 \times 10^{-3} V}{2.5 \times 10^{8}\ \frac{V}{A}} = 8.0 \times 10^{-11} A \quad \{\text{or } 80\ pA\}$$

d. Estimate the cross sectional area (A) of the conducting pathway of a Na^+ channel, in both m^2 and $Å^2$. *Assume that the length of the channel pathway is approximately the same as the thickness of the cell membrane (L), $4.0x10^{-9}$ m, and the conductivity (σ) is 0.11 $\Omega^{-1} \cdot m$.*

Ans. $$A = \rho \frac{L}{R} = \left(\frac{1\ \Omega \cdot m}{0.11}\right)^{-1} \left(\frac{4.0 \times 10^{-9}\ m}{2.5 \times 10^{8}\ \Omega}\right) = 1.45 \times 10^{-16} m^2 \quad \{\text{or } 1.45 \times 10^{4} Å^2\}$$

e. Estimate the diameter of the pore region (conducting pathway) of this protein. *Assume the conducting pathway of the ion channel is a cylinder.*

Ans. $$A = \pi r^2 = 1.45 \times 10^{4} Å^2 \Rightarrow r = \sqrt{\frac{1.45 \times 10^{4} Å^2}{\pi}} = 68.0\ Å \quad \{\text{or } 6.80 \times 10^{-9} m\}$$