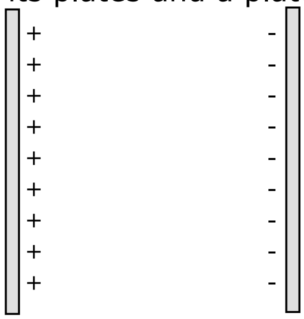
Parallel Plate Capacitors:

1. Consider parallel plate capacitor (air filled) with a surface area of 225 cm^2 and a charge of 1.5 μC (q) on each of its plates and a plate separation distance of 1.0×10^{-4} m.



a. What is the capacitance of the capacitor?

Ans. $C = \varepsilon_o \frac{A}{d} = 2.0\times10^{-9}F$

b. What is the potential difference across the capacitor?

Ans. $V = \frac{q}{C} = 750V$

c. How much energy is stored in the capacitor?

Ans. $U = \frac{1}{2}CV^2 = 5.6\times10^{-4}J$

d. If the capacitor were filled with a dielectric material, κ=3.3 (while still maintaining the same amount of charge on the plates) what is the new capacitance?

Ans. $C = \kappa C_{air} = 6.6\times10^{-9}F$

e. How much charge would be stored in this capacitor with this dielectric material, at the same potential difference as in part (b)?

Ans. $q = CV = 5.0\times10^{-6}C$

Cylindrical Capacitors:

2. A cylindrical air-filled capacitor ($R_1 = 0.5\times10^{-4}$m and $R_2 = 0.5\times10^{-2}$m) of length L=2.0 m has a potential difference of +120 V (outside – inside) between the inner and outer conductors.

a. What is the capacitance of this capacitor?

Ans. $C = \frac{2\pi\varepsilon_o L}{\ln\left(\frac{R_2}{R_1}\right)} = 2.4\times10^{-11}F$

b. How much charge is stored on each face of the capacitor?

Ans. $q = CV = 2.9\times10^{-9}C$

c. Determine the equation for the electric field vector inside the capacitor.

Ans. Apply Gauss's Law for a Gaussian cylinder (noting that R_1 & R_2 << L):

$$\Phi_E = EA_{\text{cylinder shaft}} = E(2\pi rL) = \frac{q}{\varepsilon_o} \Rightarrow \vec{E} \cong \frac{q}{2\pi rL\varepsilon_o} \left\{ \text{or } \frac{\lambda}{2\pi r\varepsilon_o} \hat{r} \right\}$$

Spherical Capacitors:

3. A spherical air-filled capacitor ($R_1 = 0.5\text{x}10^{-4}$m and $R_2 = 0.5\text{x}10^{-2}$m) has a potential difference of +120 V (outside – inside) between the surfaces.

a. What is the capacitance of this spherical capacitor?

Ans. $C = 4\pi\varepsilon_o\left(\dfrac{R_1R_2}{R_2\text{-}R_1}\right) = 5.6\text{x}10^{-15}\text{F}$

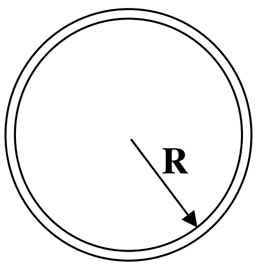
b. How much charge is stored on each face of the capacitor?

Ans. $q = CV = 6.7\text{x}10^{-13}\text{C}$

c. Determine the equation for the electric field vector inside the capacitor.

Ans. Apply Gauss's Law for a "Gaussian sphere": $\Phi_E = EA_{sphere} = E(4\pi r^2) = \dfrac{q}{\varepsilon_o} \Rightarrow \vec{E} \cong \dfrac{q}{4\pi r^2\varepsilon_o}\hat{r}$

4. Consider a spherical biological cell of radius 5.0 x 10^{-6} m. The membrane of the cell has capacitive properties no different that the electrolytic capacitors used in the lab. For this cell, the potential difference across the cell membrane (V_{in}-V_{out}) is -90 mV and the capacitance per unit surface area is 0.01 F/m^2.



a. What is the capacitance of the cell?

Ans. $C_{cell} = \left(0.01\tfrac{F}{m^2}\right)4\pi R_{cell}^2 = 3.1\text{x}10^{-12}\text{F}$

b. How much charge is stored on each face of the membrane?

Ans. $q = C_{cell}V = 2.8\text{x}10^{-13}\text{C}$

c. How much energy is stored in the cell membrane?

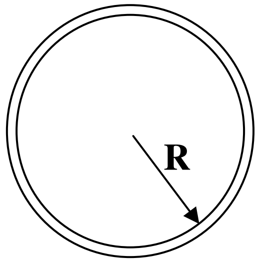
Ans. $U = \dfrac{1}{2}C_{cell}V^2 = 1.3\text{x}10^{-14}\text{J}$

d. If the dielectric constant for the cell membrane is $\kappa = 9.0$, what is the thickness of the cell membrane?

Ans. since $R_{cell} \approx R_1 \approx R_2$, the cell capacitance can be approximated by:

$$C \approx 4\pi\kappa\varepsilon_o\left(\frac{R_{cell}^2}{R_2\text{-}R_1}\right) \text{ or } d = R_2\text{-}R_1 \approx 4\pi\kappa\varepsilon_o\left(\frac{R_{cell}^2}{C}\right) = 8.0\text{x}10^{-9}\text{m}$$

5. The measured capacitance for a blood cell is measured to be $1.6\text{x}10^{-2}$ pF. Assume that the membrane thickness and the dielectric constant is the same as for the previous problem.



a. What is the surface area of the cell?

Ans. $C \approx \kappa\varepsilon_o\left(\dfrac{4\pi R_{cell}^2}{d}\right) = \kappa\varepsilon_o\left(\dfrac{A_{cell}}{d}\right) \Rightarrow A_{cell} = \dfrac{Cd}{\kappa\varepsilon_o} = 1.6\text{x}10^{-12}\text{m}^2$

b. Calculate the specific capacitance (capacitance per unit area] for this cell.

Ans. $\dfrac{C}{A_{cell}} = \; = \dfrac{1.6\text{x}10^{-14}\text{F}}{1.6\text{x}10^{-12}\text{m}^2} = 0.010\tfrac{F}{m^2}$

Capacitors in Series & Parallel

5. Two capacitors, C_1=10μF and C_2=50μF, are connected together in parallel. The potential difference across the capacitors is 100V.

a. What is the equivalent capacitance of the parallel capacitors.

Ans. $C_{eq} = C_1 + C_2 = 60\mu F$

b. How much total energy is stored in each capacitor separately and combined?

Ans. $U_{tot} = \frac{1}{2}C_{eq}V^2 = 0.3\ J$

$U_1 = \frac{1}{2}C_1V^2 = 0.05\ J$ & $U_2 = \frac{1}{2}C_2V^2 = 0.25\ J$

c. How much charge is stored in each capacitor separately and combined?

Ans. $q_{tot} = C_{eq}V = 6.0x10^{-3}\ C$

$q_1 = C_1V = 1.0x10^{-3}C$ & $q_2 = C_2V = 5.0x10^{-3}C$

6. Two capacitors, C_1=10μF and C_2=50μF, are connected together in series. The potential difference from the end of C_1 to the opposite end of C_2 is 100V.

a. What is the equivalent capacitance of the series capacitors.

Ans. $C_{eq} = \dfrac{1}{\frac{1}{C_1}+\frac{1}{C_2}} = 8.3x10^{-3}\ C$

b. How much total energy is stored in each capacitor separately and combined?

Ans. $U_{tot} = \frac{1}{2}C_{eq}V^2 = 0.042\ J$, using the charge values obtained from part c (below):

$U_1 = \dfrac{q^2}{2C_1} = 0.035\ J$ & $U_2 = \dfrac{q^2}{2C_2} = 0.0069\ J$

c. How much charge is stored in each capacitor separately and combined?

Ans. $q_{tot} = q_1 = q_2 = C_{eq}V = 8.3x10^{-4}C$