

### The Electric Potential and Potential Difference:

1. Two point charges are held at the corners of a rectangle as shown in the figure. The lengths of sides of the rectangle are 0.050 m and 0.150 m. Assume that the electric potential is defined to be zero at infinity.

a. Determine the electric potential at corner **A**.

$$\text{Ans. } V(A) = \frac{1}{4\pi\epsilon_0} \left( -\frac{5 \times 10^{-6} \text{C}}{0.150 \text{m}} + \frac{2 \times 10^{-6} \text{C}}{0.050 \text{m}} \right) = +6.0 \times 10^4 \frac{\text{J}}{\text{C}}$$



b. What is the potential difference,  $V_B - V_A$ , between corners **A** and **B**?

$$\text{Ans. } V(B) = \frac{1}{4\pi\epsilon_0} \left( -\frac{5 \times 10^{-6} \text{C}}{0.050 \text{m}} + \frac{2 \times 10^{-6} \text{C}}{0.150 \text{m}} \right) = +7.8 \times 10^5 \frac{\text{J}}{\text{C}}$$

$$\Delta V = V(B) - V(A) = -7.8 \times 10^5 \frac{\text{J}}{\text{C}} - 6.0 \times 10^4 \frac{\text{J}}{\text{C}} = -8.4 \times 10^5 \frac{\text{J}}{\text{C}}$$

c. What is the electric potential energy of a  $+3 \mu\text{C}$  charge placed at corner **A**?

$$\text{Ans. } U(A) = qV(A) = (3.0 \times 10^{-6} \text{C})(6.0 \times 10^4 \frac{\text{J}}{\text{C}}) = 0.18 \text{ J}$$

2. Consider 2 concentric charged conducting spheres,  $R_1 = 0.1 \text{ m}$  and  $R_2 = 0.2 \text{ m}$  respectively. The electric potential of the inner sphere is  $+4 \text{ V}$  and the outer sphere is  $-6 \text{ V}$ .

a. Determine the equation for the electric potential for the following regions associated with the spheres:

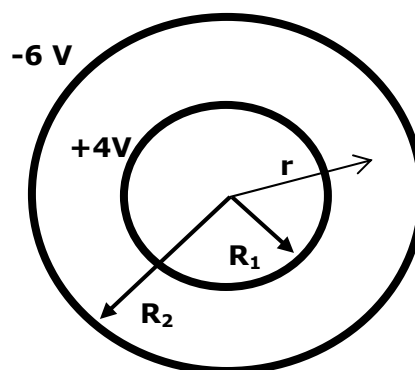
i)  $r < R_1$ :

Ans. Since  $E = 0$ ,  $V = \text{constant} = +4 \text{ V}$

ii)  $R_1 < r < R_2$

Ans. Since  $V$  transitions from  $+4$  to  $-6 \text{ V}$ ,  $\Delta V_{R_1 \rightarrow R_2} = -10 \text{ V}$ . Applying Gauss's Law to obtain the form of  $E(r)$ :

$$|\vec{E}(r)| = \left( \frac{1}{4\pi\epsilon_0} \right) \frac{q_{\text{inner}}}{r^2}$$



The electric potential referenced from  $r = R_1$  is:

$$\Delta V = V - V_{\text{inner}} = -\int_{\infty}^r E(r) dr = -\left( \frac{1}{4\pi\epsilon_0} \right) \int_{\infty}^r \frac{q_{\text{inner}}}{r^2} dr = \left( \frac{q_{\text{inner}}}{4\pi\epsilon_0} \right) \left( \frac{1}{r} - \frac{1}{R_1} \right)$$

Since  $V_{\text{outer}}$  is  $-10 \text{ V}$  lower than the center of the sphere, we can substitute an equivalent set of values for  $\Delta V$  which will be easier to work with:  $V_{\text{inner}}' = +10 \text{ V}$  and  $V_{\text{outer}}' = 0 \text{ V}$  ( $\Delta V_{R_1 \rightarrow R_2} = 0 \text{ V} - 10 \text{ V} = -10 \text{ V}$ ). The shifted electric potential ( $V'$ ) then becomes:

$$V'_{\text{inner}} = \left( \frac{q_{\text{inner}}}{4\pi\epsilon_0 R_1} \right) = +10 \text{ V} \Rightarrow V'(r) = \left( \frac{q_{\text{inner}}}{4\pi\epsilon_0 r} \right) - 10 \text{ V}$$

Substituting the original potential values leads to:  $V(r) = \left( \frac{q_{\text{inner}}}{4\pi\epsilon_0 r} \right) - 16 \text{ V}$

**Quick Check:**

(1) When  $r = R_1 = 0.01 \text{ m}$ :  $V(R_1) = \left( \frac{q_{\text{inner}}}{4\pi\epsilon_0 R_1} \right) - 16 \text{ V} = +4 \text{ V}$ . Solving for the total

charge on the inner sphere:  $q_{\text{inner}} = 2.22 \times 10^{-11} \text{ C}$

(2) When  $r = R_2 = 0.02 \text{ m}$  and  $q_{\text{inner}} = 2.22 \times 10^{-11} \text{ C}$ :

$V(R_2) = \left( \frac{q_{\text{inner}}}{4\pi\epsilon_0} \right) \left( \frac{1}{R_2} \right) - 16 \text{ V} = -6 \text{ V}$ . The equation checks for  $R_1 < r < R_2$ .

iii)  $r > R_2$

Ans. According to Gauss' Law, the total electric flux is equal to the enclosed charge for any closed surface. Since the charge has the same magnitude on each sphere,  $q_{\text{enclosed}} = 0$  for a closed sphere where  $r < R_2 \rightarrow E = 0$  outside sphere 2 and therefore:

$V(r) = \text{constant} = -6 \text{ V}$  (assuming there are no additional stray charges around...).

b. Determine the equation for the electric field for the following regions associated with the spheres:

i)  $r < R_1$

Ans. Since there is no enclosed charge for any Gaussian surface inside of the inner sphere, application of Gauss's Law leads to  $E = 0$ .

i)  $R_1 < r < R_2$

Ans. The electric field is related to the spatial derivative of the electric potential:  $\vec{E} = -\frac{dV}{dr} \hat{r}$

Using the equation for the electric potential above:  $\vec{E}(r) = -\frac{d}{dr} \left( \frac{q_{\text{inner}}}{4\pi\epsilon_0 r} - 10 \text{ V} \right) \hat{r} = \frac{q_{\text{inner}}}{4\pi\epsilon_0 r^2} \hat{r}$ .

Alternatively, Gauss' Law could have been applied directly.

i)  $r > R_2$

Ans. Since  $V(r) = \text{constant} = -6 \text{ V}$  outside the outer sphere, there is no electric field ( $E=0$ ).

c. What is the electric potential at point P, located 1 m from the center of the spheres?

Ans. Since  $V(r) = \text{constant}$  outside the outer sphere,  $V(P) = -6 \text{ V}$ .

d. How much work is needed to take an electron at infinity and place it at point P?

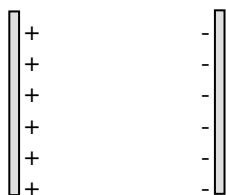
Ans.  $W_{\infty \rightarrow P} = -\Delta U = -q_{\text{electron}} V(P) = -9.6 \times 10^{-19} \text{ J}$ .

e. What is the surface charge density for the each of the spheres?

Ans. Sphere 1:  $\sigma_1 = \frac{q}{4\pi R_1^2} = 1.8 \times 10^{-8} \frac{\text{C}}{\text{m}^2}$ .

Sphere 2:  $\sigma_2 = \frac{q}{4\pi R_2^2} = 4.4 \times 10^{-9} \frac{\text{C}}{\text{m}^2}$

3. Consider a charged parallel plate capacitor (air filled) with a charge of  $1.5 \mu\text{C}$  ( $q$ ) on each of its plates and a plate distance of  $1 \times 10^{-4} \text{ m}$ . The potential difference between the plates is 120 V.



a. Calculate the magnitude and direction of the electric field between the plates.

$$\text{Ans. } \vec{E} = -\frac{dV}{dx}\hat{i} = -\frac{\Delta V}{\Delta x}\hat{i} = 1.2 \times 10^6 \frac{\text{N}}{\text{C}}\hat{i}$$

b. What is the difference in electric potential energy between an electron placed very close to the left plate compared to the right plate?

$$\text{Ans. } \Delta U = q_{\text{electron}}\Delta V = 1.9 \times 10^{-17} \text{ J}.$$

Note, the total potential energy difference between the plates for all charge already present is

$$\text{equal to: } \Delta U = q\Delta V_{\text{avg}} = (1.5 \times 10^{-6} \text{ C})\left(\frac{120 \text{ V}}{2}\right) = 9.0 \times 10^{-5} \text{ J}$$

4. A fixed point charge,  $q_1 = -2.0 \times 10^{-6} \text{ C}$  is located in space.

a. What is the electric potential difference between infinity (i.e. a long way away) and a position 0.1 m away from  $q_1$ ?

$$\text{Ans. } V = \frac{q}{4\pi\epsilon_0 r} = -1.8 \times 10^5 \text{ V}$$

b. How much work is required to bring a second charge,  $q_2 = -1.0 \times 10^{-6} \text{ C}$  from infinity to a position 0.1m away from  $q_1$ ?

$$\text{Ans. } W = -qV = -(-1.0 \times 10^{-6} \text{ C})(-1.8 \times 10^5 \text{ V}) = -0.18 \text{ J}$$

c. How much work is required to bring  $q_2$  from 0.1m to 0.05m away from  $q_1$ ?

$$\text{Ans. } W = -q\Delta V = -(-1.0 \times 10^{-6} \text{ C})[-3.6 \times 10^5 \text{ V} + 1.8 \times 10^5 \text{ V}] = -0.18 \text{ J}$$

d. If both  $q_1$  and  $q_2$  are fixed at a separation distance of 0.05m, at what position is the electric potential equal to zero?

Ans. Electric potentials add together for individual charges as scalars not vectors. Since both charges have the same sign, there is no position where the absolute potential is zero (except infinity)...

5. A proton (fixed) and an electron (initially at rest but free to move) are located alone and separated by a great distance (but are even farther away from anything else).

a. How much work does the proton perform to "pull" the electron to a position of  $5.29 \times 10^{-11} \text{ m}$  away from the proton?

$$\text{Ans. } W = \frac{e^2}{4\pi\epsilon_0 r} = 4.4 \times 10^{-18} \text{ J}$$

- b. Apply the Conservation of Energy, determine the kinetic energy of the electron when it is located  $5.29 \times 10^{-11}$  m from the proton.

Ans.  $W = -\Delta U = \Delta K = 4.4 \times 10^{-18} \text{ J}$

- c. Estimate how fast the electron is moving when it gets there?

Ans.  $\Delta K = K = \frac{1}{2}mv^2 = 4.4 \times 10^{-18} \text{ J} \Rightarrow v = 3.1 \times 10^6 \frac{\text{m}}{\text{s}}$

6. The electric potential difference across the outer membrane of a biological cell (thickness = 6.0 nm) is 70 mV. The inside of the cell is more negative than the outside. When an "ion channel" embedded in the cell membrane is activated a water-filled pathway is opened and ions can flow through the channel.

- a. What is the magnitude of the electric field inside this ion channel?

Ans.  $|\vec{E}| = -\frac{\Delta V}{\Delta x} = 1.2 \times 10^6 \frac{\text{N}}{\text{C}}$

- b. How much electrical work would be performed on a  $\text{Na}^+$  ion as it is "pulled" through the channel? Assume that only the electric potential drives the ion across the membrane.

Ans.  $W = -q\Delta V = 1.1 \times 10^{-20} \text{ J}$

- c. How many  $\text{Na}^+$  ions would cross the channel if the total charge movement from outside to inside is 25 pC?

Ans.  $N = \frac{q}{e} = 1.6 \times 10^8 \text{ Na}^+ \text{ ions}$

- d. How much electrical work would be performed on to drive all of the  $\text{Na}^+$  ions in (c) through the channel? Assume that the electric potential difference across the cell remains constant.

Ans.  $W = -q\Delta V = 1.8 \times 10^{-12} \text{ J}$

- e. Starting from rest, how fast would you expect a  $\text{Na}^+$  ion to be traveling when it reaches the other end of the ion channel? Assume that the  $\text{Na}^+$  ion encounters no resistance and all potential energy is converted into work.

Ans.  $\Delta K = W = \frac{1}{2}mv^2 = 1.1 \times 10^{-20} \text{ J} \Rightarrow v = 760 \frac{\text{m}}{\text{s}}$