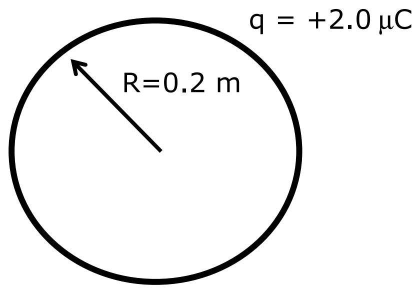
Gauss' Law:

1. Consider a hollow spherical conductive shell of radius (R) 0.2 m with a fixed charge of +2.0 μC uniformly distributed on its surface.



a) What is the electric field at all points inside the sphere? Express your answer as a function of the distance (r) from the center of the sphere.

Ans. For a Gaussian sphere, r<R: $\Phi_E = E\Delta A = \dfrac{q_{enclosed}}{\varepsilon_o} = 0 \Rightarrow E = 0$

b) What is the electric field outside the sphere? Express your answer as a function of the distance (r) from the center of the sphere.

Ans. For a Gaussian sphere, r>R: $\Phi_E = E\Delta A = E(4\pi r^2) = \dfrac{q}{\varepsilon_o} \Rightarrow \left|\vec{E}\right| = \left(\dfrac{1}{4\pi\varepsilon_o}\right)\dfrac{q}{r^2}$

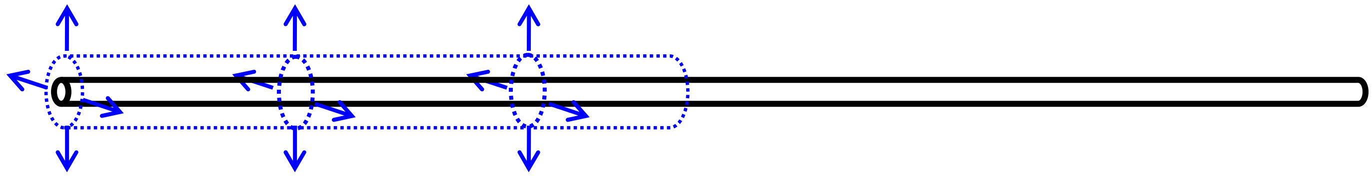
c) What if the sphere is a solid conductive sphere? What is the electric field at all points inside the sphere? Express your answer as a function of the distance (r) from the center of the sphere.

Ans. For a Gaussian sphere, r<R: $\Phi_E = E\Delta A = E(4\pi r^2) = \dfrac{q_{enclosed}}{\varepsilon_o} = \dfrac{\rho(\frac{4}{3}\pi r^3)}{\varepsilon_o} \Rightarrow \left|\vec{E}\right| = \dfrac{\rho r}{3\varepsilon_o}$

d) What is the electric field at all points outside the sphere? Express your answer as a function of the distance (r) from the center of the sphere.

Ans. For a Gaussian sphere, r>R: $\Phi_E = E\Delta A = E(4\pi r^2) = \dfrac{q}{\varepsilon_o} \Rightarrow \left|\vec{E}\right| = \left(\dfrac{1}{4\pi\varepsilon_o}\right)\dfrac{q}{r^2} = \dfrac{\sigma R^3}{3\varepsilon_o r^2}$

2. Consider a "line of charge", where the charge per unit length is λ (in C/m) and the line is "infinitely long".
   a) Draw the line of charge and attempt to draw the E field lines around the line.



   b) Apply Gauss's Law to the line of charge and determine the magnitude of the electric field at all points away from the line? Express your answer as a function of the distance (r) from the line of charge.

Ans. For a Gaussian cylinder, charge density λ, length l & radius r:

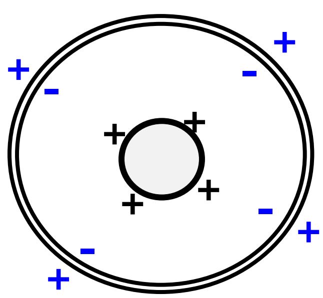
$$\Phi_E = E_1A_1 + E_2A_2 + E_3A_3 = E(2\pi rl) = \frac{q_{enclosed}}{\varepsilon_o} = \frac{\lambda l}{\varepsilon_o} \Rightarrow \left|\vec{E}\right| = \left(\frac{1}{2\pi\varepsilon_o}\right)\frac{\lambda}{r}$$

   c) If the surface charge density is 0.5×10^{-3} C/m, what is the magnitude of the electric field at a distance of 0.2 m from the line of charge?

Ans. $\left|\vec{E}\right| = \left(\frac{1}{2\pi\varepsilon_o}\right)\frac{\lambda}{r} = 4.5\times10^{7}\,\frac{N}{C}$

3. Consider two concentric conductive spheres (of radius r_1 and r_2, respectively). The inner sphere has a uniform surface charge (q) of +2.0 μC.

a) Draw in the charged regions of the 2 spheres on the diagram. (Remember that charged is induced in the conductive outer sphere)



b) What is the magnitude of the electric field at the following regions?

   i) $r < r_1$

Ans. $\left|\vec{E}\right| = 0$

   ii) $r_2 > r > r_1$

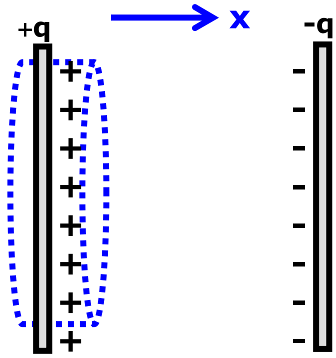
Ans. $\left|\vec{E}\right| = \left(\frac{1}{4\pi\varepsilon_o}\right)\frac{q}{r^2}$ where q = +2.0 μC

   iii) $r > r_2$

Ans. $\left|\vec{E}\right| = \left(\frac{1}{4\pi\varepsilon_o}\right)\frac{q}{r^2}$ where q = +2.0 μC

4. The plates of a parallel plate capacitor have a uniform charge of +1.0 μC and -1.0 μC, respectively. Each plate is a square with sides of length of 0.5 m.

a. Apply Gauss' Law to the capacitor to determine the magnitude of the electric field between the plates.



Ans. For a Gaussian pillbox:

$$\Phi_E = EA_{\text{inner face}} = E(\pi r^2) = \frac{q_{enclosed}}{\varepsilon_o} \Rightarrow \left|\vec{E}\right| = \frac{q_{enclosed}}{\pi r^2 \varepsilon_o} = \frac{\sigma}{\varepsilon_o} = 4.5\text{x}10^5 \tfrac{N}{C}$$

b. What is the electric force exerted on an electron placed inside the plates of the capacitor?

Ans. $\vec{F} = e\vec{E} = \left(e\frac{\sigma}{\varepsilon_o}\right)\hat{i} = -\left(1.6\text{x}10^{-19}C\right)\left(4.5\text{x}10^5 \tfrac{N}{C}\right)\hat{i} = -7.2\text{x}10^{-14}\ N\ \hat{i}$

c. What is the acceleration of the electron while it is in between the capacitor plates?

Ans. $a = \frac{\vec{F}}{m_e} = \left(\frac{-7.2\text{x}10^{-14}\ N}{9.1\text{x}10^{-31}\ kg}\right)\hat{i} = -7.9\text{x}10^{16} \tfrac{m}{s^2}\ \hat{i}$

d. For a 0.2 m plate separation, calculate the final speed of an electron (initially at rest), that is placed at the negative charged plate and just reaches the positive plate.

Ans. $v = -\left(\sqrt{2ad}\right)\hat{i} = -\left(\sqrt{2\left(7.9\text{x}10^{16} \tfrac{m}{s^2}\right)(0.2m)}\right)\hat{i} = -1.8\text{x}10^8 \tfrac{m}{s}\ \hat{i}$